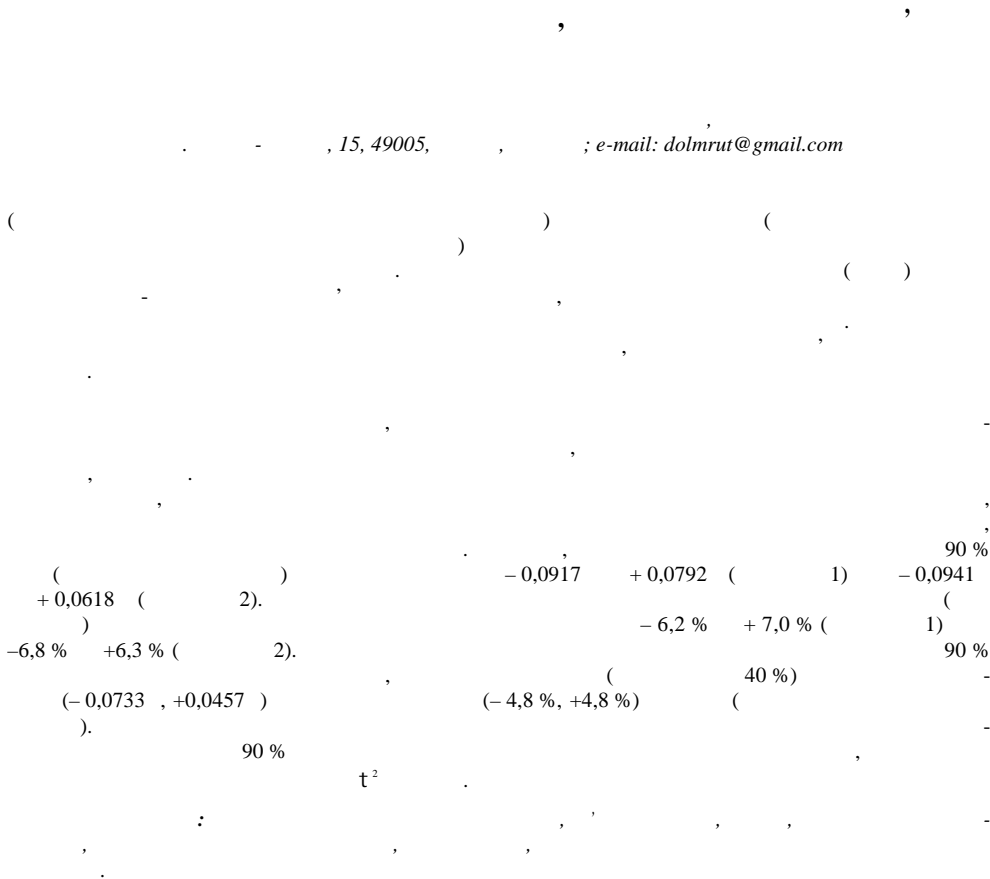


C. .



The thrust spread of a stand-alone rocket engine caused by external (the pressure and temperature of the propellant components at the engine inlet) and internal (spread in the geometry and operating conditions of the engine units and assemblies) factors is known from experimental tests or can be computed by a known procedure. As a rule, liquid-propellant propulsion systems (LPPSs) of launch vehicle lower stages include a cluster of several engines, whose thrust spread cannot often be determined from firing tests due to limited capabilities of bench equipment. The aim of this work is to develop an approach to determining the thrust spread of an LPPS comprising a cluster of two and more engines.

For a multiengine propulsion system, this methodological approach also includes the development of a mathematical model of engine interaction in an LPPS and calculations of an LPPS startup at different combinations of spread in the external and internal factors in cases where the parameter spreads of all engines are both identical and different.

For an LPPS with two engines and a common oxidizer feed pipeline, the paper gives an example of calculating the effect of external and internal factors on the thrust spread of each engine and the LPPS as a whole during an LPPS startup. It is shown that the calculated spread of the 90 percent thrust (combustion chamber pressure) time lies in the range -0.0917 s to $+0.0792$ s (engine 1) and -0.0941 s to $+0.0618$ s (engine 2). The calculated variations of the combustion chamber pressure (engine thrust) from its nominal value lie in the range -6.2 percent to $+7.0$ percent (engine 1) and -6.8 percent to $+6.3$ percent (engine 2). The calculated spreads of the 90 percent thrust time and the thrust for the LPPS as a whole are far smaller (about by 40 percent) and lie in the range -0.0733 s to $+0.0457$ s for the time and -4.8 percent to $+4.8$ percent for the thrust (about the nominal thrust). Using Pearson's chi-squared test, an estimate is obtained for the goodness of fit of the anticipated theoretical distributions of the 90 percent thrust time spread and the steady thrust spread to the obtained statistical ones both for the two engines and for the LPPS as a whole.

Keywords: liquid-propellant rocket propulsion system, engine cluster, startup, mathematical simulation, external and internal factors, thrust spread, goodness of fit of a theoretical distribution to a statistical one.

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 [1], [2]. -
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 , [3]: -
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 , [4]. -
 [4]. -
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 , [5], [6].
 [5], [7] -
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 [10]. () () [11], -
 [12], [13]) -
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 , [4], -
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[14].

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1.

[4].

[15], [16].

[5], [6],

[17], [18],

[8], [9], [10],

[1],

[5], [7]

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[19] – [21].

. 1.

[22], [23].

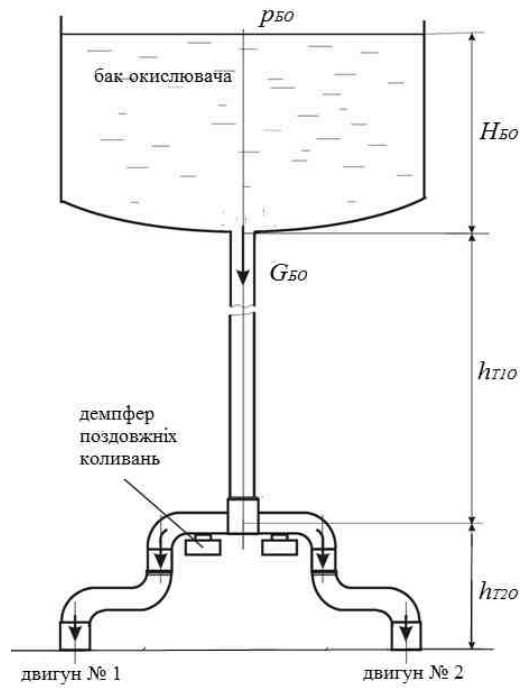
$$\begin{cases} \frac{\partial p}{\partial z} + \frac{1}{g \cdot F} \cdot \frac{\partial G}{\partial t} + \frac{k}{g \cdot F} \cdot G = 0, \\ \frac{\partial G}{\partial z} + \frac{g \cdot F}{c^2} \cdot \frac{\partial p}{\partial t} = 0, \end{cases}$$

$p, G -$
 $F -$

; $z -$
; $k -$

; g –

; c –



[24],

. 1 –

$$o = u_1 + a_{U1} G^2 - \chi (H_o + h_1) + J_{U1} \frac{dG}{dt},$$

$$C_U \frac{dp_{U1}}{dt} = G - \sum_{i=1}^2 G_{1i} - \sum_{i=1}^2 G_{Di},$$

$$u_1 = u_1 + a_{U2} G_1^2 - \chi h_2 + (J_{U2} + J) \frac{dG_1}{dt},$$

C_U –

; a_{U1} , a_{U2} , J_{U1} , J_{U2} –

; H_o , h_1 , h_2 –

; G –

(); o , $1i$, G_{1i} –

i – ; G_{Di} –

i –

2.

[4]. :

1. , , - , ().

1 -

		, %	1	2
1		$\pm 3,9$	1	12
2		$\pm 3,4$	2	13
3		± 2	3	14
4	± 5	4	15
5		± 2	5	16
6	$\pm 1,3$	6	17
7	I	± 2	7	18
8 I	± 3	8	19
9	-	± 20	9	20
10		± 10	10	21
11	-	± 20	11	22

[4]

[25].

2 3

[25].

n -

x_i

n -

x_i

(x_1, x_2, \dots, x_n)

	τ									
	1					2				
	X ₁	X ₂	X ₃	...	X ₁₁	X ₁₂	X ₁₃	X ₁₄	...	X ₂₂
1	0,5	0,5	0,5	...	0,5	0,5	0,5	0,5	...	0,5
2	0,25	0,75	0,25	...	0,75	0,25	0,75	0,25	...	0,75
3	0,75	0,25	0,75	...	0,25	0,75	0,25	0,75	...	0,25
4	0,125	0,625	0,875	...	0,625	0,875	0,875	0,125	...	0,375
5	0,625	0,125	0,375	...	0,125	0,375	0,375	0,625	...	0,875
6	0,375	0,375	0,625	...	0,375	0,625	0,125	0,375	...	0,625
7	0,875	0,875	0,125	...	0,875	0,125	0,625	0,875	...	0,125
8	0,063	0,938	0,688	...	0,063	0,188	0,438	0,563	...	0,188
9	0,563	0,438	0,188	...	0,563	0,688	0,938	0,063	...	0,688
10	0,313	0,188	0,938	...	0,813	0,438	0,688	0,813	...	0,938
11	0,813	0,688	0,438	...	0,313	0,938	0,188	0,313	...	0,438
12	0,188	0,313	0,313	...	0,688	0,813	0,563	0,688	...	0,313
...
255	0,996	0,004	0,77	...	0,332	0,043	0,941	0,996	...	0,793
256	0,002	0,502	0,908	...	0,803	0,455	0,115	0,689	...	0,037

	τ			
	-	-	-	-
	X ₂₃	X ₂₄	X ₂₅	X ₂₆
1	0,5	0,5	0,5	0,5
2	0,25	0,75	0,25	0,75
3	0,75	0,25	0,75	0,25
4	0,875	0,875	0,125	0,375
5	0,375	0,375	0,625	0,875
6	0,625	0,125	0,375	0,625
7	0,125	0,625	0,875	0,125
8	0,188	0,438	0,563	0,313
9	0,688	0,938	0,063	0,813
10	0,438	0,688	0,813	0,563
11	0,938	0,188	0,313	0,063
12	0,813	0,563	0,688	0,188
...
255	0,582	0,098	0,949	0,84
256	0,979	0,541	0,389	0,588

3.

4 90 %
 () -
 , -0,0879 +0,0749 .
 (-
) -6,2 % +7,0 %
 (. .5).

4- 90 %
 (min - , max -)

		min	max
1		-0,0879	+0,0749
2	, 1	-0,0917	+0,0792
3	, 2	-0,0941	+0,0618
4	,	-0,0733	+0,0457

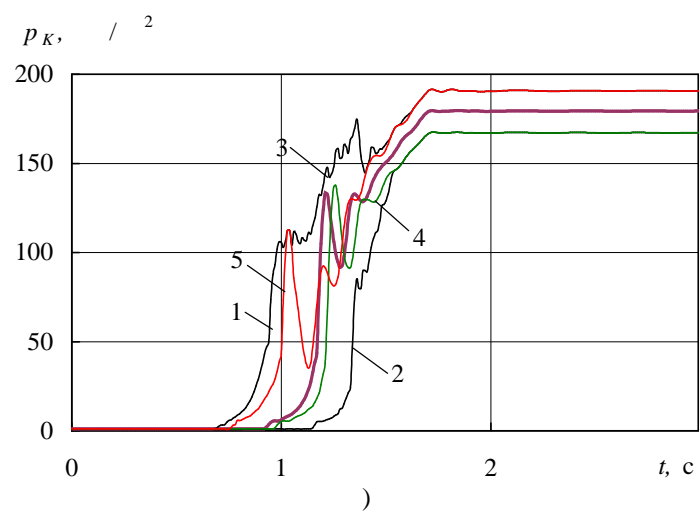
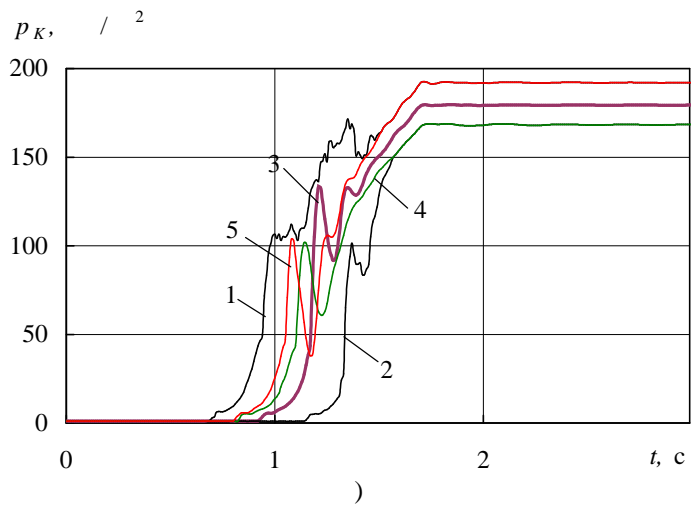
5- ()
 (min - , max -)

		, %	
		min	max
1		-6,2	+7,0
2	, 1	-6,2	+7,0
3	, 2	-6,8	+6,3
4	,	-4,8	+4,8

90 %
 176 () 191 ().
 176 ()
) 247 ().
 4 5
 90 % ()
 ,
 ,
 90 %
 1) -0,0941 +0,0618 (-0,0917 +0,0792 (-
 2).)
 ()
 (. .5) -6,2 %
 +7,0 % (1) -6,8 % -6,3 % (2).
 ,
 90 %
 (-0,0733 , +0,0457), (40 %),
 (. .4).
 ()
 40 %)
 . .5,

() . -4,8% +4,8%

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1 2



.2 -
1 () 2 ()

1, 2 - ; 3 - ; 4 - ; 5 -

.3

(247).

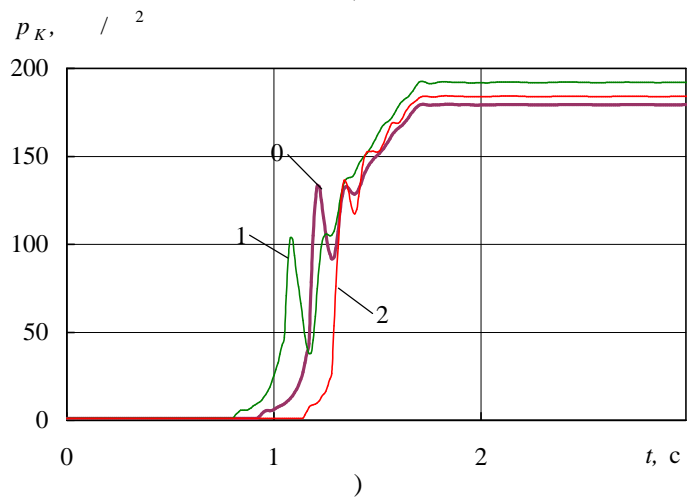
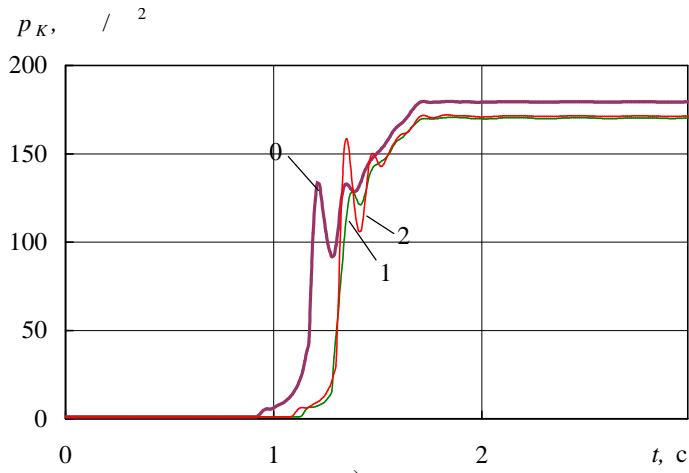
(72)

164 173

176 247

1

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1 2

(t²)

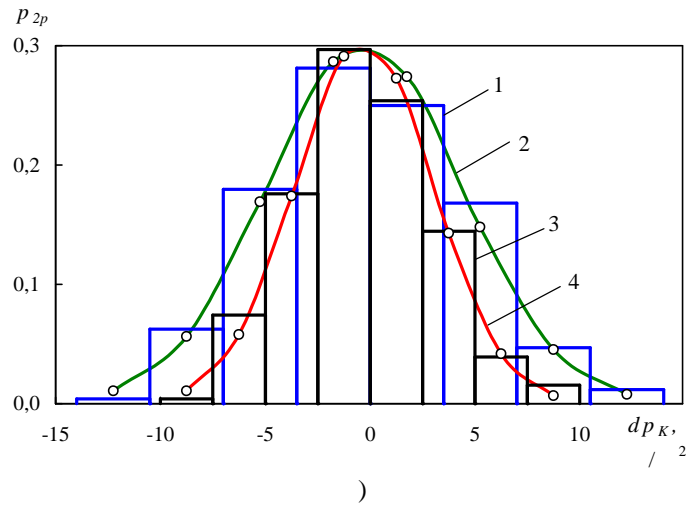
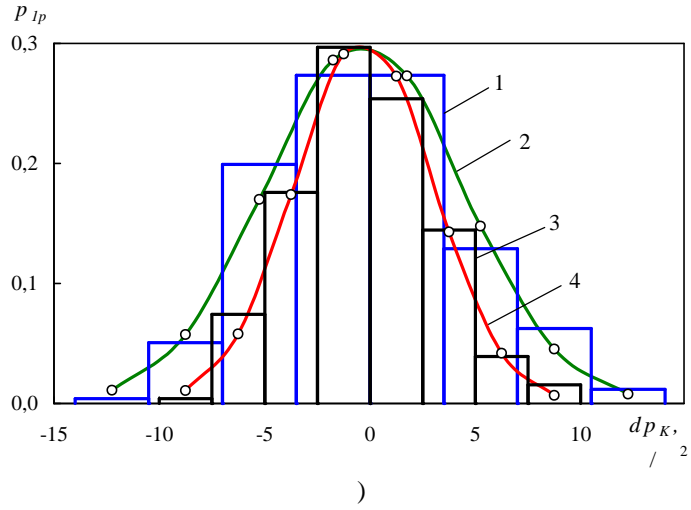
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\bar{M}

t^2 ,
 t^2 .



.4 -

1 () 2 () (1, 2) -
 (3, 4): 1, 3 - ; 2,

4 -

t^2 $t^2 = 11,1$.
 t^2

– 21 / ².

– 10,2 / ².

90 %

90 % () – 0,0917
+ 0,0792 (1) – 0,0941 + 0,0618 (2).

()
– 2 % + 7,0 % ()

1) – 6,8 % + 6,3 % (2).

90 %

(– 0,0733 , + 0,0457) (40 %)
(– 4,8 % , + 4,8 %) ()

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90 %

t²

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533 .

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