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MATHEMATICAL MODELING OF PROBE MEASUREMENTS IN A SUPERSONIC FLOW OF A FOUR-COMPONENT COLLISIONLESS PLASMA

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Метою роботи є розробка процедури відновлення концентрації і температури електронів, іонного складу плазми за вольтамперною характеристикою (ВАХ) ізольованої зондової системи з циліндричними електродами. Плазма чотирьохкомпонентна, складається з електронів, іонів двох сортів з істотно різними масами і нейтралів. Вимірювальний зонд і опорний електрод зондової системи можуть бути складені з декількох циліндрів. Електроди зондової системи поперечно обтікаються надзвуковим потоком низькотемпературної беззіштовхувальної плазми з відомою масовою швидкістю.

З використанням відомих теоретичних і експериментальних залежностей іонного й електронного струмів на циліндр побудована математична модель збирання струму для ізольованої зондової системи при довільному відношенні площ електродів. Модель включає розрахунок рівноважного потенціалу опорного електрода при зміні напруги зсуву зонда. Розроблено процедуру ідентифікації локальних параметрів плазми з залученням апріорної інформації про властивості плазми й умови експерименту. Досліджено вплив концентрації і температури електронів, іонного складу плазми на зондовий струм ізольованої зондової системи при різному відношенні площ електродів, що збирають струм. Визначено області потенціалів зсуву зонда і значення відносини площ електродів, що відповідають найбільшому і найменшому впливу параметрів, що відновлюються, на зондовий струм. Отримані кількісні обмеження на потенціали зсуву і відношення площ використані в схемі зондових вимірювань і при побудові цільової функції порівняння теоретичної апроксимації зондового струму з результатами вимірювання ВАХ.

Виконано числове моделювання зондових вимірювань в умовах іоносфери. Підтверджено працездатність процедури ідентифікації локальних параметрів чотирьохкомпонентної плазми за електронною гілкою ВАХ ізольованої зондової системи. Отримані результати можуть бути використані в діагностиці іоносферної плазми на понадмалих супутниках.

Ключові слова: *іони плазми двох сортів, ізольована зондова система з циліндричними електродами, математична модель збирання струму, параметрична ідентифікація, апріорна інформація*.

The aim of this work is the development of a procedure for extracting the plasma electron density and temperature and ion composition from the current-voltage characteristic (C –V characteristic) of an isolated probe system of cylindrical electrodes. The plasma is four-component and consists of electrons, ions of two species with significantly different masses, and neutrals. The measuring probe and the reference electrode of the probe system may be made up of several cylinders. The electrodes of the probe system are placed transversely to a supersonic flow of a low-temperature collisionless plasma with a specified mass velocity.

Using the familiar theoretical and experimental relationships for the ion and electron currents to a cylinder, a mathematical model of current collection is constructed for an isolated probe system at an arbitrary ratio of the electrode surface areas. The model includes the calculation of the equilibrium potential of the reference electrode as a function of the probe bias voltage. A procedure is developed for the identification of local plasma parameters using a priori information on the plasma properties and the experimental conditions. The effect of the electron density and temperature and the ion composition on the probe current of the isolated probe system at different ratios of the current-collecting electrode surface areas is studied. The ranges of the probe bias potentials and the values of the electrode surface area ratio that maximize and minimize the effect of the sought-for parameters on the probe current are determined. The quantitative restrictions on the bias potentials and the surface area ratio obtained in this study are used in the probe measurement procedure and in the objective function for comparing the theoretical approximation of the probe current with the measured I– characteristics.

A numerical simulation of probe measurements under the ionospheric conditions was conducted to verify the efficiency of the procedure for extracting the local parameters of a four-component plasma from the electron branch of the I –V characteristic of an isolated probe system. The results obtained may be used in ionospheric plasma diagnostics onboard nanosatellites.

Keywords*: plasma ions of two species, isolated probe system with cylindrical electrodes, mathematical model of current collection, parametric identification, a priori information*.

Introduction. Diagnostics of ionospheric plasma on spacecraft is traditionally carried out using electric probes. Due to the simplicity of the apparatus and the acceptable measurement accuracy, electric probes are successfully used to study the kinetic parameters of charged components of the ionospheric plasma since the first rocket launches [1].

The basic elements of the probe measuring system are the measuring electrode (the probe) and the reference electrode, which have electrical contact with the plasma and have the area of the current-collecting surface of S_p and S_{cp}, respectively. The classical theory of a single cylindrical probe is applicable if the ratio of the collecting surface areas satisfies a rather strict condition $S_s = S_{cp}/S_p \geq 10^4$, [1, 2]. Application of the single Langmuir probe scheme for ionospheric measurements most often supposes the spacecraft body (outer surfaces of the conductive elements that are not insulated from plasma) to be the reference electrode. Very low density of the ionospheric plasma and the trend towards a decrease in the spacecraft size (development of micro- and nanosatellites) make it difficult to satisfy the condition $S_s \geq 10^4$ for the current-collecting electrodes of the measuring scheme of a single Langmuir probe.

The theory of current collection by a cylindrical Langmuir probe in a supersonic flow of a rarefied weakly ionized gas is developed well for a three-component plasma containing ions of the same kind. The Earth's ionosphere is a volatile multicomponent medium. The ionic composition of the ionosphere varies significantly depending on the time (day / night variations), season (winter / summer variations), and intensity of solar and geomagnetic activity. Fig. 1 shows the dependence of the main ionic constituents of the ionosphere (oxygen O^+ – solid curves, nitrogen N^+ – dashed curves, hydrogen H^+ – dotted curves) on the altitude in the daytime (a) and at night (b) in winter 01.01.2019 (thick curves) and in summer 01.07 .2019 (thin curves). This data is obtained using IRI-2012 model [3] for location Lat = 50° N, Long = 40° E.

One can see from Fig. 1,a) that in the daytime under the considered conditions, the fraction of relatively lightweight atomic hydrogen ions does not exceed 10 %, and for probe diagnostics, the approximation of the "average mass" of ions can be used. At night (Fig. 1,b)) in winter for altitude of more than 600 km, the fraction of atomic hydrogen ions can exceed 50 %, and, taking into account the aforementioned restriction for a single Langmuir probe scheme, probe diagnostics based on the three-component plasma theory (considering ions of only one kind) requires additional substantiation.

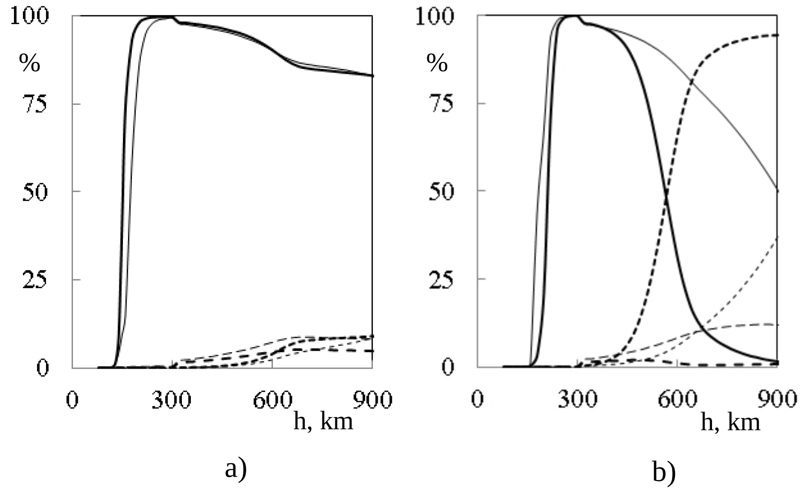


Fig. 1

Previously in [4] an effective procedure was developed for interpreting the I -V characteristic of a single cylindrical probe in a three-component collisionless plasma flow consisting of neutrals, positive ions, and electrons. The procedure is based on parametric identification of the I –V characteristic of a probe transversely placed in plasma flow using the a priori information on the plasma properties and experimental conditions. Later, this procedure was extended [5] to an isolated probe system with cylindrical electrodes at an arbitrary ratio of the surface areas of the probe and the reference electrode. The developed procedure allows one to interpret probe measurements in the three-component plasma approximation on nanosatellites. In [6], the procedure for interpreting probe measurements [4, 5] was adapted for diagnostics of laboratory four-component plasma containing atomic and molecular ions of the working gas of a plasma source. Using an isolated probe system with sectional cylindrical electrodes, the procedure allows restoring plasma parameters, including the degree of dissociation, provided that the squares of the mass velocity of atomic and molecular ions in the source jet are inversely proportional to their masses. The current article substantiates the possibility of using the procedure [4, 5] for diagnosing a high-speed flow of four-component plasma of an arbitrary ionic composition by the electronic branch of the I –V characteristic of an isolated probe system with sectional cylindrical electrodes.

Formulation of the problem. Let us consider a model of a probe measuring system with cylindrical electrodes, placed transversely in a supersonic flow of collisionless plasma with a mass velocity V. It's assumed that the base radii of the probe r_p and the reference electrode r_{cp} are significantly less than their length, the end surfaces of the electrodes are isolated from the plasma, the electrostatic and gas-dynamic influence of the electrodes on each other in the plasma is small, and there is no emission current from the electrode surfaces. The plasma is four-component (it consists of neutrals, positive singly charged ions of two species and electrons), quasi-neutral, its flow around the electrodes is collisionless, the influence of the magnetic field on the probe current is negligible, the velocity distribution of particles of the same species in unperturbed plasma is Maxwellian.

We assume that for the radii of the probe r_p and the reference electrode r_{cp} the restrictions [4, 5] satisfy:

$$r_p / \lambda_d \le 1, \; r_{cp} / \lambda_d < 10, \tag{1}$$

where λ_d – Debye length in unperturbed plasma.

The ion composition of the plasma is characterized by the parameter $\chi_n = n_{i,1} / (n_{i,1} + n_{i,2}) \equiv n_{i,1} / n_e$, where $n_{i,1}$, $n_{i,2}$ are the densities of ions of the species $i,1$, $i,2$, respectively, n_e is the electron density (condition of plasma quasineutrality follows $n_{i,1} + n_{i,2} = n_e$). The temperatures of the ions of the kinds $i,1$ and $i,2$ are assumed to be equal, $T_{i,1} = T_{i,2} = T_i$.

Mathematical model of current collection. For the formulated above problem, the main parameters of the unperturbed plasma are: the density of electrons n_e, the part of ions of the species $i,1$ among positively charged particles χ_n, the temperature of electrons T_e, ion temperature T_i and masses $m_{i,1}$, $m_{i,2}$. Let's presume that ions $i,2$ are heavier: $m_{i,1} < m_{i,2}$.

Let's make use of the electric current approximation to a cylinder transversely placed in collisionless plasma [4, 5], obtained on the basis of the classical asymptotic Langmuir relations [7], analytical studies [8] and calculations [4, 9, 10]. Assuming that the presence of different ion species in a supersonic plasma flow does not lead to a significant change in the self-consistent electric field in the vicinity of the immersed cylinder [6], the dimensionless total current on a cylinder under a potential φ related to the unperturbed plasma potential (electron current is positive) is:

$$\bar{I}_c(\varphi) = \bar{I}_e(\varphi) - \chi_n \sqrt{\chi_m \mu/\beta} \cdot \bar{I}_{i,1}(\varphi) - (1-\chi_n)\sqrt{\mu/\beta} \cdot \bar{I}_{i,2}(\varphi), \quad S_i > \sqrt{\chi_m}, \qquad (2)$$

$$\bar{I}_e(\varphi) = \begin{cases} 2/\sqrt{\pi} \cdot \sqrt{\pi/4 + \varphi}, & \varphi > 0; \\ \exp(\varphi), & \varphi \le 0 \end{cases},$$

$$\bar{I}_{i1}(\varphi) = \begin{cases} \sqrt{2/\pi} \exp\left(-\beta\varphi + S_i^2/\chi_m\right), & \beta\varphi \ge S_i^2/\chi_m; \\ 2/\sqrt{\pi} \cdot \sqrt{1/2 + S_i^2/\chi_m - \beta\varphi}, & \beta\varphi < S_i^2/\chi_m \end{cases},$$

$$\bar{I}_{i,2}(\varphi) = \begin{cases} \sqrt{2/\pi} \exp\left(-\beta\varphi + S_i^2\right), & \beta\varphi \ge S_i^2; \\ 2/\sqrt{\pi}\sqrt{1/2 + S_i^2 - \beta\varphi}, & \beta\varphi < S_i^2 \end{cases},$$

where $\bar{I}_c$, $\bar{I}_e$ are respectively the total and electron currents on a cylinder, both normalized by the chaotic electron current; $\bar{I}_i$ is the ion current, normalized by the chaotic current of ions of the corresponding kind, $\chi_m = m_{i,2}/m_{i,1}$ is the different ion kinds mass ratio, $\mu = m_e/m_{i,2}$ is the charged particles mass ratio, $\beta = T_e/T_i$ is the temperature ratio, $S_i = V/u_{i,2}$ is the ionic velocity ratio for heavier ions $i,2$. The dimensionless potential φ is normalized by kT_e/e, where k is the Boltzmann constant, e is the elemental charge. The thermal current of particles of the kind α is $I_{\alpha,0} = j_{\alpha,0} \cdot S_c$, where $j_{\alpha,0} = en_\alpha u_\alpha / 2\sqrt{\pi}$ is the thermal current density, $u_\alpha = \sqrt{2kT_\alpha/m_\alpha}$ is the thermal velocity, T_α and m_α is the particle temperature and mass, S_c is the area of the cylinder collecting surface. Subscript $\alpha = i$ refers to ions, $i,1$ – lightweight ion species, $i,2$ – heavier ion species, e – electrons.

Direct problem of probe measurements. The experimental I -V characteristic is the dependence of measured current I_p in the "probe – plasma – reference electrode" circuit on the bias potential of the probe U_{iz} relative to the potential of the reference electrode U_{cp}. Probe potential U_p with respect to undisturbed plasma is $U_p = U_{iz} + U_{cp}$.

The characteristic time of establishing equilibrium potential in the ionosphere is rather short [1]. Therefore, the considered isolated probe system is always under an equilibrium potential determined by the zero total current of charged particles through all collecting surfaces of the electrodes. The equilibrium potential of the reference electrode U_{cp}, that corresponds to the bias voltage U_{iz}, is derived from the current balance equation [5]

$$S_s \cdot \bar{I}_{cp}(\varphi_{iz}) + \bar{I}_p(\varphi_{iz}) = 0 . \qquad (3)$$

Here, the dimensionless currents to the reference electrode $\bar{I}_{cp}(\varphi_{iz}) = \bar{I}_c(\varphi_{cp})$ and to the probe $\bar{I}_p(\varphi_{iz}) = \bar{I}_c(\varphi_{iz} + \varphi_{cp})$ are determined by relations (2). Solving for each bias potential φ_{iz} the nonlinear equation (3) for the reference electrode potential φ_{cp}, we find the dependence of the reference electrode equilibrium potential on the probe bias potential, $\varphi_{cp} = \Phi(\varphi_{iz})$.

Thus, the dimensionless I -V characteristic of the probe writes

$$\bar{I}_p(\varphi_{iz}) = \bar{I}_c(\Phi(\varphi_{iz}) + \varphi_{iz}),$$

in a dimensional form:

$$I_p(U_{iz}) = j_{e0} \cdot S_p \cdot \bar{I}_c(\Phi(eU_{iz}/kT_e) + eU_{iz}/kT_e). \qquad (4)$$

Since the dependence of the electric current on the electrode potential and parameters χ_n, χ_m, μ, β, S_i is a continuous function (2), the solution of the nonlinear equation (3) exists, it is unique for all considered values of the bias potential φ_{iz} and it can be found using iterative method [4, 5].

In the case of one ion species ($\chi_n = 0$) and sufficiently large positive probe potential relative to the plasma, $\varphi_p = \varphi_{iz} + \varphi_{cp} >> 1$, the current balance equation (3) has an analytical solution [6]

$$\varphi_{cp} = -\frac{(\pi/4 + \varphi_{iz}) - S_s{}^2 \mu/\beta \cdot (1/2 + S_i{}^2)}{1 + S_s{}^2 \mu}.$$

From physical considerations $\varphi_{cp} < 0$, then we obtain the restriction to the bias potential

$$\varphi_{iz} > \varphi_{iz}^*(S_s) = S_s^2 \mu/\beta \cdot (1/2 + S_i^2) - \pi/4 . \qquad (5)$$

The dimensionless I -V characteristic of the probe in the electron saturation region $\varphi_{iz} > \varphi_{iz}^*$ in this case is determined as follows

$$\bar{I}_p(\varphi_{iz}) \approx \frac{2}{\sqrt{\pi}} \cdot S_s \cdot \sqrt{\mu/(1 + S_s{}^2 \mu)} \cdot \sqrt{(1/2 + S_i{}^2)/\beta + (\pi/4 + \varphi_{iz})} . \qquad (6)$$

As one can see, in contrast to a single Langmuir probe case, the electron saturation current depends on the ion flow velocity S_i and the degree of plasma nonisothermality β. This is due to the insulation of the probe system, as well as the shape and plasma flow pattern around the reference electrode. The range of the applicability of the obtained approximation of the probe electron saturation current (6) is determined by condition (5). An increase in the electrode areas ratio S_s leads to an increase in the bias voltage φ_{iz} required to reach electron saturation.

Relations (2), (3), which determine the parametric representation of the I –V characteristic of the "probe – plasma – reference electrode" system in the case of two ions species, include dimensionless parameters χ_n, χ_m, μ, β, S_i, S_s, φ_{iz}, determined through the parameters of the unperturbed quasi-neutral plasma, the probe and the reference electrode: n_e, T_e, $m_{i,1}$, $m_{i,2}$, $n_{i,1}$, T_i, V, S_p, S_{cp}, U_{iz}.

Fig. 2 shows dependences of the probe current $\bar{I}_p$ (a) and probe equilibrium potential $\varphi_p = \varphi_{iz} + \varphi_{cp}$ (b) on the bias potential φ_{iz} for various ratio of the electrode areas S_s = 50, 100, 150, 200, 300, 400. For each S_s there are four thin curves: calculations for χ_n = 0.1 (solid), χ_n = 0.3 (long dashed), χ_n = 0.6 (short dashed), χ_n = 0.9 (dotted). The thick solid curve is calculated at S_s = 1000, χ_n = 0. Calculations is performed for the parameters $S_i = 4.9$, $\mu = 1.7 \cdot 10^{-5}$, $\beta = 1.3$, that correspond to the ionospheric plasma at an altitude of about 700 km [4].

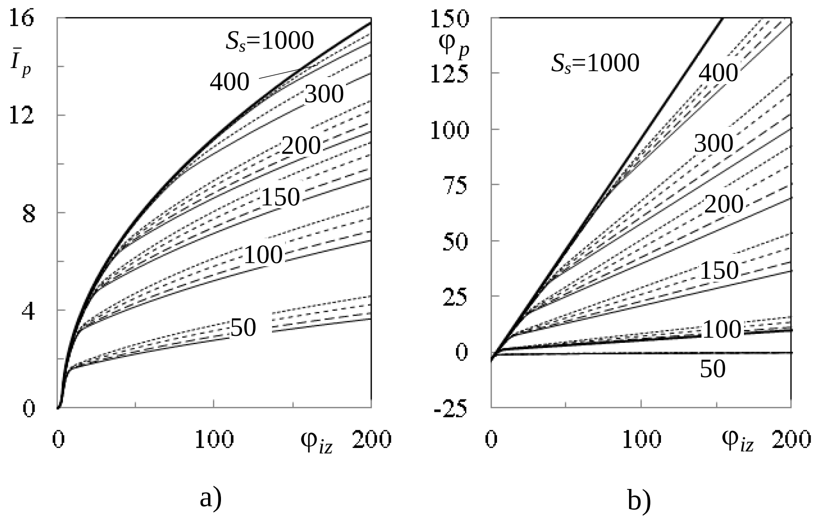


Fig. 2

An increase in ratio of the electrode areas S_s leads to an increase in the electron saturation current and in the probe equilibrium potential relative to the plasma potential. At each value of the parameter S_s, an increase in χ_n (part of lighter ions in plasma) leads to an increase in the probe current. The influence of the parameter χ_n on the collected electron current and the equilibrium probe potential is observed at Fig. 2 only in the region of electron saturation. In the transition region of the I –V characteristic of the probe system, the influence of the parameter χ_n is insignificant. The performed calculations show that the dimensionless bias potential $\varphi_{iz} \approx \varphi_{iz}^*(S_s)$, where $\varphi_{iz}^*(S_s)$ is determined in (5), separates the electronic branch of the I –V characteristic into the transition region and the electron saturation region

Inverse problem of probe measurements. To apply relations (2), (3) to the interpretation of probe measurements in the ionosphere, following [4, 5], we use the geometric parameters of the isolated probe system S_p, S_s and the parameters of the plasma flow

$$n_e,\ T_e,\ \chi_n,\ \beta,\ \chi_m,\ \mu,\ V\,. \qquad (7)$$

Let the base radii of the probe r_p and the reference electrode r_{cp} be specified and satisfy the constraints (1). We assume that the ion masses (parameters χ_m, μ) and the mass velocity of the plasma flow V are given.

Calculations performed for the flow of a four-component plasma, similarly to the case of a three-component plasma [4, 5], revealed a weak dependence of the collected probe current on the degree of plasma nonisothermality β at all values of the ratio of the electrode surface areas S_s and bias potential U_{iz}. This represents a well-known drawback of probe diagnostics – the ion temperature ($T_i = T_e/\beta$) cannot be reliably determined from the probe characteristic in high-speed nonisothermal ($T_i \neq T_e$) plasma flow [2, 10]. Following [4, 5], we fix the value of the parameter β based on the a priori information about the valid degree of plasma non-isothermality. Such information can be obtained from the IRI-2012 model [3] for the ionosphere or from additional independent measurements of the ion temperature in the nonisothermal plasma [4].

The geometric parameters S_p, S_s of the current-collecting electrodes of the probe system must be selected to ensure reliable determination by the electronic branch of the I –V characteristic of the density n_e and electron temperature T_e, as well as the fraction of lightweight ions χ_n in the supersonic flow of a four-component plasma.

Let G denote the vector of parameters (7), then formally the relation for the probe current (4), including the calculated relations (2), (3), can be written as follows:

$$I_p(U_{iz}, S_s, G) = j_{e0}(G) \cdot S_p \cdot \bar{I}_c\big(\varphi_p(U_{iz}, S_s, G), G\big),$$

$$\varphi_p(U_{iz}, S_s, G) = eU_{iz}/kT_e + \Phi\big(eU_{iz}/kT_e, S_s, G\big),$$

where $\varphi_p(U_{iz}, S_s, G)$ is the dimensionless potential of the probe relative to the plasma corresponding to the dimensional bias potential U_{iz} and the values of parameters (7) for the probe system with the area ratio S_s; $\Phi(\varphi_{iz}, S_s, G)$ is the root of the nonlinear equation (3) corresponding to the dimensionless bias potential φ_{iz} and values of parameters (7) for the probe system with the area ratio of S_s.

Let $I_{ex}(U_{iz}, S_s)$ be the experimentally obtained I -V characteristic of the proposed probe system with the area ratio S_s. The inverse problem of probe measurements is to determine the unknown parameters from the set (7), at which the theoretical I -V characteristic best fits the experimental one. In general case, the problem of identifying the plasma parameters (7) is stated as follows [4]:

$$G^*:\ F(G^*) = \min_{G \in D} F(G), \quad F(G) = \left\| I_p(U_{iz}, S_s, G) - I_{ex}(U_{iz}, S_s) \right\|_{M_{iz}}. \qquad (8)$$

Here $\|f(U_{iz})\|_{M_{iz}}$ is the discrete quadratic norm of the grid function $f(U_{iz})$, the set M_{iz} defines the nodes of the grid function, D is the set of valid values of G: $G^{\min} \le G \le G^{\max}$, where $G^{\min}$, $G^{\max}$ are the vectors of the smallest and largest values of parameters (7), respectively. The set of valid values of parameters (7) is determined from physical considerations and a priori information about the studied plasma flow [4].
For example, a priori information on parameters χ_m and μ can be obtained as long as the chemical composition of the plasma is given, on V – from the space-craft trajectory, on n_e, T_e, χ_n, β – from previous measurements in similar conditions.

Eq. (2), (4), combined with Eq. (3), determine the probe current as a piecewise-analytical function of the potential U_{iz} and all the parameters (7). Its smoothness breaks at regions that satisfy

$$eU_{iz}/kT_e + \Phi(eU_{iz}/kT_e, S_s, G) = 0,$$

$$eU_{iz}/kT_e + \Phi(eU_{iz}/kT_e, S_s, G) = m_e/kT_e \cdot V^2/2\mu\chi_m,$$

$$eU_{iz}/kT_e + \Phi(eU_{iz}/kT_e, S_s, G) = m_e/kT_e \cdot V^2/2\mu.$$

For a probe system with electrodes surface areas ratio of S_s, the above equalities determine the surfaces of the space of variables (U_{iz}, G) on which the derivatives of the function $I_p(U_{iz}, S_s, G)$ have discontinuities of the first kind. At all other points of the set D function $I_p(U_{iz}, S_s, G)$ is analytic, hence continuously differentiable with respect to all its variables. The variational problem (8) can be solved by methods based on Newton's method with constraints on the parameters.

Analysis of problem parameters. Let g be one of parameters from the set (7). For the function $I_p(U_{iz}, S_s, G)$ in the vicinity of a certain point $G_0 = (n_{e0}, T_{e0}, \chi_{n0}, \beta_0, \chi_{m0}, \mu_0, V_0)$ we define the dimensionless function of sensitivity to parameter g:

$$\bar{I}_{p,g}(U_{iz}, S_s, G_0) = \frac{g_0}{j_{e0} \cdot S_p} \frac{\partial I_p(U_{iz}, S_s, G_0)}{\partial g}.$$

Small changes in parameters (7) $g = g_0(1+\varepsilon_g)$ in the vicinity of the point G_0 leads to a change in the dimensionless probe current

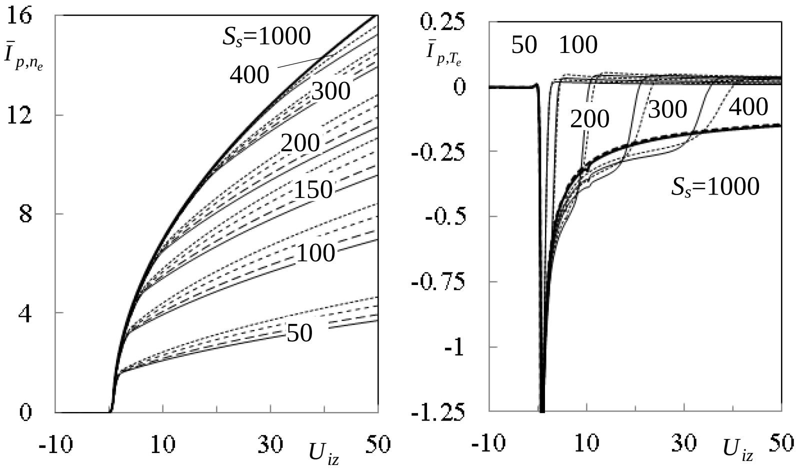
$$\bar{I}_p(U_{iz}, S_s, G) \approx \bar{I}_p(U_{iz}, S_s, G_0) + \sum_g \varepsilon_g \bar{I}_{p,g}(U_{iz}, S_s, G_0).$$

On the basis of these relations, the analysis of the influence of parameters (7) on the I –V characteristic of the proposed isolated probe system was carried out at various reference electrode to probe area ratios during ionospheric studies at an altitude of about 700 km [4].

The calculations show that for the considered model "probe – four-component plasma – reference electrode", the dependence of plasma parameters n_e, T_e, β,

μ, V on the probe current qualitatively and quantitatively corresponds to the dependence obtained in [5] for the three-component plasma model. Under the electrodes area ratio $S_s > 500$ for density n_e, mass ratio μ, temperature ratio β the sensitivity functions are as well qualitatively and quantitatively correspond to the ones obtained in [4] for the model of a single Langmuir probe. In contrast to the model of a single electric probe, for the proposed isolated probe system the sensitivity functions to the flow velocity V and electron temperature T_e change their sign.

For the proposed probe current collection model (2) – (4), Fig. 3 represents the results of calculating the sensitivity functions to the density n_e (a) and electron temperature T_e (b) vs. the probe bias potential U_{iz}, in Volts, for different electrodes surface areas ratio S_s. For each value of S_s in Fig. 3a there are four thin curves calculated at the values of the parameter χ_n = 0.1, 0.3, 0.6, 0.9, in Fig. 3b – two thin curves calculated at values of χ_n = 0.1, 0.9. Curves are labeled as in Fig. 1. The thick solid curve is calculated at S_s = 1000, χ_n = 0.

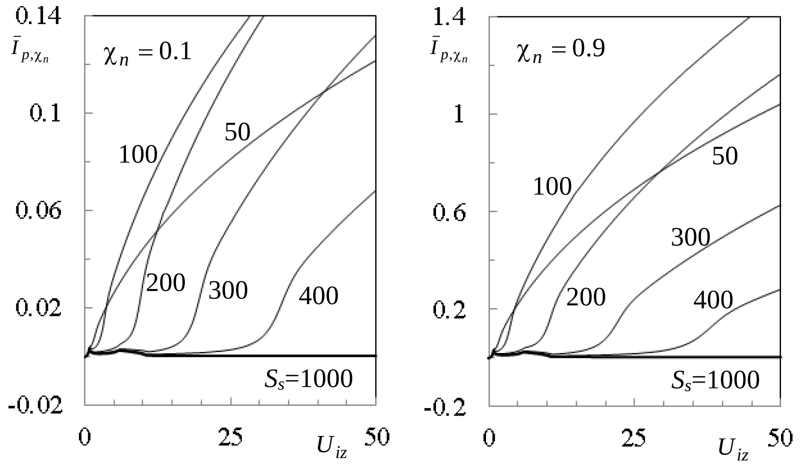


a) b)

Fig. 3

From Fig. 3,a) one can see that, similarly to the case of single Langmuir probe, the electron density n_e has a dominant effect on the probe current in the region of electron saturation at all values of the area ratio S_s. Fig. 3,b) shows that with an increase in the area ratio S_s, the region of influence of T_e spreads towards greater probe bias potential U_{iz}. Under the area ratio of S_s >500, the influence of the parameter T_e on the probe current practically does not change with further increase in the area of the collecting surface of the reference electrode.

Just like in the theory of a single Langmuir probe, parameter T_e significantly influence on the probe current only in the transitional part of the I –V characteristic, near the floating potential of the probe system. The performed calculations showed that the region of influence of the electron temperature T_e on the probe current is estimated by $0 < U_{iz} < 1.6 \cdot \varphi_{iz}^{*}(S_s) \cdot kT_e/e$ where the dimensionless potential $\varphi_{iz}^{*}(S_s)$ is determined in (5).



a) b)

Fig. 4

Fig. 4 shows the results of calculating the function of sensitivity to the parameter χ_n at the values of χ_n = 0.1 (a) and χ_n = 0.9 (b) vs. the probe bias potential U_{iz}, in Volts, for different values of the areas ratio S_s. An increase in the area ratio S_s shifts the region of influence of the parameter χ_n towards higher potential U_{iz} for all valid values of χ_n. So, at S_s >400 in the transition region of the I -V characteristic, where the influence of the electron temperature T_e is strong, the influence of the parameter χ_n on the collected current is minimal. The calculations showed that the region of influence of the parameter χ_n on the probe current collection is well described by condition (5). An increase in the probe bias potential U_{iz} increases the value of the sensitivity function $\bar{I}_{p,\chi_n}$. The strongest sensitivity of the collected current to the parameter χ_n is observed at $S_s \in [100, 200]$. An increase in the parameter χ_n leads to an increase in the values of the sensitivity function $\bar{I}_{p,\chi_n}$, and, consequently, to better accuracy of identification of the parameter χ_n by solving the variational problem (8).

Probe measurement procedure. On the basis of the results of the performed analysis, let's construct a procedure for identifying the parameters n_e, T_e, χ_n, by the electronic branch of the I –V characteristic of an isolated probe system. Let the reference electrode be a series of parallel cylinders, and each of them can be connected or disconnected from the electrical measurement circuit. Such a measuring probe system makes it possible to measure the I –V characteristic at different values of the area ratio S_s. Let S_s^{*} and S_s^{**} be the values of such area ratios ($S_s^{*} < S_s^{**}$). We assume that the local flow parameters remain for different reference electrode area. Then, to each bias potential φ_{iz} in the measuring system with

$S_s = S_s^*$ corresponds the probe current I_p^*, and I_p^{**} in the measuring system with $S_s = S_s^{**}$.

The value S_s^* must be in the range [100, 200] to ensure the strongest sensitivity of the collected probe current I_p^* to the plasma ion composition. The value S_s^{**} must satisfy the condition $S_s^{**} > 400$, and then in the transition region of I -V characteristic, where the strongest sensitivity of the probe current to the electron temperature T_e is observed, the influence of the parameter χ_n on the collected current is negligible small. The use of two I -V characteristics $I_p^*(U_{iz})$ and $I_p^{**}(U_{iz})$ separates the regions of influence of the parameter χ_n (ion composition of the plasma) and the electron temperature T_e on the probe current. In this case, the objective function of the variational problem (8) is the sum over $S_s = S_s^*, S_s^{**}$ of the norms of the difference between the theoretical and experimental I -V characteristics. The mesh M_{iz} of bias potential values U_{iz} for the objective function must cover the transition region and the electron saturation region, $0 < U_{iz} < U_{iz}^{**}$, where $U_{iz}^{**} \geq 2 \cdot \varphi_{iz}^*\left(S_s^{**}\right) \cdot kT_e/e$, and $\varphi_{iz}^*(S_s)$ is defined in (5).

The results of the numerical simulation of the identifying the values of the parameters n_e, T_e, χ_n by solving the variational problem (8) at the electrodes surface area ratio of S_s^*= 100, 150, S_s^{**} = 400, 500, 600 confirmed the possibility of diagnosing a four-component plasma using the proposed isolated probe system. Similarly to the case of dissociated gas in the plasma of a jet [6], an increase in the ratio S_s^{**}/S_s^* improves the accuracy of identifying ion composition of the plasma χ_n.

Conclusions. A mathematical model of current collection in a flow of four-component plasma for a probe system with cylindrical electrodes with an arbitrary ratio of the probe and the reference electrode areas is constructed. The model includes the calculation of the equilibrium potential of the reference electrode when the bias voltage of the probe changes. On the basis of the constructed model, a procedure was developed for calculating the theoretical I –V characteristic of a cylindrical probe in a supersonic flow of low-temperature collisionless plasma and for identifying the local plasma parameters using a priori information on the plasma properties and experimental conditions.

The influence of the electron density and temperature, ion composition on the probe current of an isolated probe system is investigated for different ratio of the electrode areas. The regions of the probe bias potential and the area ratio corresponding to the strongest and the weakest influence of the sought-for parameters on the probe current are determined. The obtained quantitative restrictions on the bias potential and the area ratio allow constructing a scheme of probe measurements, in which the regions of influence of the ion composition and the electron temperature on I –V characteristic are separated. Numerical simulation of probe measurements in ionospheric conditions is performed. The efficiency of the procedure for identifying the local parameters of four-component plasma by the electronic branch of an isolated probe system is confirmed.

The obtained results can be used in the planning and interpretation of experiments on diagnostics of ionospheric plasma.

1. *Boyd R. L. F.* Langmuir probes on spacecraft, in Plasma Diagnostics, Ed. By Lochte-Holtgreven. North-Holland Publ. Co., Amsterdam, 1968. P. 732–776.
2. *Chung, P. M., Talbot L., Touryan K. J.* Electric Probes in Stationary and Flowing Plasmas. Springer-Verlag, 1975. 150 p. https://doi.org/10.1007/978-3-642-65886-0
3. Online resource – https://ccmc.gsfc.nasa.gov/modelweb/models/iri2012_vitmo.php.
4. *Лазученков Д. Н., Лазученков Н. М.* Интерпретация зондовых измерений в потоке бесстолкновительной плазмы. Технічна механіка. 2018. № 1. С. 107–120. https://doi.org/10.15407/itm2018.01.107
5. *Лазученков Д. Н., Лазученков Н. М.* Моделирование восстановления кинетических параметров ионосферной плазмы по вольтамперной характеристике цилиндрического зонда. Технічна механіка. 2019. № 4. С. 107–118. https://doi.org/10.15407/itm2019.04.107
6. *Lazuchenkov D. N., Lazuchenkov N. M.* Determination of parameters of a dissociated supersonic rarefied plasma flow by current-voltage characteristics of isolated system of cylindrical probes. Технічна механіка. 2020. № 2. С. 80–88. https://doi.org/10.15407/itm2020.02.080
7. *Mott-Smith H., Langmuir I.* The theory of collectors in gaseous discharges. Phys. Rev. 1926. V. 28. № 5. P. 727–763. https://doi.org/10.1103/PhysRev.28.727
8. *Hoegy W. R., Wharton L. E.* Current to a moving cylindrical electrostatic probe. Journal of Applied Physics. 1973. V. 44, No. 12. P. 5365–5371. https://doi.org/10.1063/1.1662157
9. *Laframboise J. G.* Theory of Spherical and Cylindrical Langmuir Probes in a Collisionless Maxwellian Plasma at Rest. Report No. 100. Univ. of Toronto, Institute of Aerospace Studies. 1966. 210 p. https://doi.org/10.21236/AD0634596
10. *Godard R., Laframboise J.* Total current to cylindrical collectors in collision less plasma flow. Space Science. 1983. V. 31, № 3. P. 275–283. https://doi.org/10.1016/0032-0633(83)90077-6

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