

**Numerical gas-dynamic computational methods in problems of rarefied jet flow about obstacles**

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The development of competitive space hardware calls for continuing improvements in the accuracy of simulation of gas-dynamic processes in the space vehicle vicinity. This may contribute to extending the active life of spacecraft, thus improving the economic efficiency of space activities. In particular, quite a topical problem is the simulation of the interaction of rarefied jets from the propulsion system of a spacecraft with its individual components. To solve this problem in the case of a rather high surrounding vacuum, use is made of the molecular-kinetic concept of the gas structure based on the Boltzmann equation. The aim of this paper is to overview existing methods of simulation of gas-dynamic processes near spacecraft in a rarefied gas flow with account for propulsion system jets and to choose the most promising approaches to the solution of this problem. Among the methods considered, several main lines are set off: approximate, analytical, and numerical methods. Approximate methods use physical models of jet flow, approximation of numerical results, or a combination of both approaches. Analytical methods are based on essentially simplified assumptions and are intended for a very narrow class of problems. Numerical methods are the most universal tool of theoretical study. At the same time, each numerical method has a range of application of its own. At present, the most used and promising methods are statistical simulation methods: the direct simulation Monte Carlo method (DSMCM) and the test particle method (TPM). The former splits the continuous process of molecule motion and collisions in a rarefied gas into two successive independent stages (free-molecular transfer and relaxation) at each small time step. The simulation is done by time steps and in fact reproduces a nonstationary process. The latter, the TPM, consist in a statistical successive simulation of the wandering of test particles (molecules) on the background of field ones about the cells of the computational grid. Test particles, which move within the cells of the computational area, periodically collide with the obstacle in the flow and field particles, and in doing so they gradually change both their velocity and the field characteristics. For both statistical approaches, the simulation accuracy, as can be expected, is inversely proportional to the square root of the number of tests: the number of time steps and modeling particles for the DSMCM and the number of successively simulated test particle trajectories for the TPM. This may greatly affect the possibility of attaining a desired accuracy.

**Keywords:** *gas-dynamic processes, analytical and numerical methods, rarefied jet flow, statistical simulation methods, direct simulation Monte Carlo method, test particle method.*

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