

## Self-vibrations of a truncated conical sandwich shell with a honeycomb core made by additive technologies

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This paper presents a nonlinear mathematical model of self-vibrations of conical sandwich shells with a honeycomb core made by additive technologies. The vibrations of the structure are described by fifteen unknowns. Each layer of the structure is described by five unknowns: three projections of the displacements of the layer middle surface and two rotation angles of the middle surface normal. Displacement continuity conditions at the layer interfaces are used. The higher-order shear theory is used to describe the stress-strain state of the structure. The case of conical sandwich shell – supersonic gas flow interaction is considered. Due to this interaction, self-vibrations of the shell structure are set up. In their analysis, the geometrical nonlinearity of the structure is accounted for. Motion equations of the structure are derived using the assumed-mode method, which uses the kinetic and the potential energy of the structure. The self-vibrations are represented as eigenmode expansions, which contain a set of generalized coordinates. A system of nonlinear autonomous ordinary differential equations in the generalized coordinates is derived. The self-vibrations are studied using a combination of the shooting technique and the parameter continuation method. Multipliers are calculated to analyze the stability of periodic vibrations and their bifurcations. The dynamic instability of the structure's trivial equilibrium is studied by numerical simulation. For clamped-clamped and cantilever shells, the properties of their periodic, quasiperiodic, and chaotic motions are analyzed in detail.

**Keywords:** nonlinear dynamical system, conical sandwich shell, self-sustained vibrations, quasiperiodic vibrations, chaos.

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