

JACOBI PROBABILITY DISTRIBUTION FOR APPROXIMATION OF EMPIRIC STATISTIC DISTRIBUTIONS

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The paper purpose is to demonstrate the opportunities of the Jacobi probability distributions for fitting the statistical populations. The universal Pearson and Johnson systems of the distributions, the generalized lambda distribution and the Gram-Charlier distribution, which are widely used for fitting statistical populations, are analyzed. It is pointed out that the main disadvantage of these distributions is that they do not take into account real limited ranges of variations in the random variables. The paper considers the theoretical problems of the construction of the one-dimensional Jacobi probability distribution, based on the expansion the unknown density function in the term of the system of the orthogonal Jacobi polynomials with variations in a limited interval. The optimality principles of the Jacobi distribution are formulated to approximate the statistical data, and practical recommendations are given for its construction. In particular, the best fitting results are obtained for the Jacobi distribution constructed with the ultraspherical orthogonal Jacobi polynomials. The application of the Jacobi distribution is determined, which is significantly wider than the application of the Gram-Charlier distribution. Methods for determining the limited points of the Jacobi distribution are presented. Examples demonstrate the advantages of the Jacobi distribution for fitting the statistical populations in comparison with the universal distributions used in practice.

Keywords: fitting statistic data, Jacobi distribution, Gram-Charlier distribution, generalized lambda-distribution, region of variations in random value.

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