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## VERIFICATION OF A MATHEMATICAL MODEL FOR THE SOLUTION OF THE STEFAN PROBLEM USING THE MUSHY LAYER METHOD

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The use of solar energy has limitations due to its periodic availability: solar plants do not operate at night and are ineffective in dull weather. The solution of this problem involves the introduction of energy storage and duplication systems into the conversion loop. Among the energy storage systems, solid–liquid phase transition modules have significant energy, ecologic, and cost advantages. Physical processes in modules of this type are described by a system of non-stationary nonlinear partial differential equations with specific boundary conditions at the phase interface. The verification of a method for solving the Stefan problem for a heat-storage material is presented in this paper.

The use of the mushy layer method made it possible to simplify the classical mathematical model of the Stefan problem by reducing it to a nonstationary heat conduction problem with an implicit heat source that takes into account the latent heat of transition. The phase transition is considered to occur in an intermediate zone determined by the solidus and liquidus temperatures rather than in an infinite region. To develop a Python code, use was made of an implicit computational scheme in which the solidus and liquidus temperatures remain constant and are determined in the course of numerical experiments.

The physical model chosen for computer simulation and algorithm verification is the process of ice layer formation on a water surface at a constant ambient temperature. The numerical results obtained allow one to determine the temperature fields in the solid and the liquid phase and the position of the phase interface and calculate its advance speed.

The algorithm developed was verified by analyzing the classical analytical solution of the Stefan problem for the one-dimensional case at a constant advance speed of the phase interface. The value of the verification coefficient was determined from a numerical solution of a nonlinear equation with the use of special built-in Python functions. Substituting the data for the physical model under consideration into the analytical solution and comparing them with the numerical simulation data obtained with the use of the mushy layer method shows that the results are in close agreement, thus demonstrating the correctness of the computer algorithm developed.

These studies will allow one to adapt the Python code developed on the basis of the mushy layer method to the calculation of heat storage systems with a solid-liquid phase transition with account for the features of their geometry, the temperature level, and actual boundary conditions.

**Keywords:** *mathematical simulation, numerical algorithm, Stefan problem, mushy layer method, analytical solution, verification.*

1. Liu M., Saman W., Brun F. Review on storage materials and thermal performance enhancement techniques for high temperature phase change thermal storage systems. *Renewable and Sustainable Energy Reviews*. 2012 V.16. No. 4. Pp, 2118 - 2132.

<https://doi.org/10.1016/j.rser.2012.01.020>

2. Lissner M., Tissot J., Leducq D., Azzouz K., Fournaison L. Performance study of latent heat accumulators: Numerical and experimental study, *Applied Thermal Engineering*, 2016. V. 102. Pp. 604-614.

<https://doi.org/10.1016/j.applthermaleng.2016.03.011>

3. Knysh L. I. Simulation of energy transfer processes in a solid-liquid heat accumulator of a space power plant. System Design and Analysis of Aerospace Technique Characteristics. 2014. V. 17. Pp. 89-95. (in Russian).

4. Knysh L. I. Mathematical simulation of heat exchange processes in solid-liquid heat storage systems. Industrial Heat Engineering. 2014. V. 36. No. 4. Pp. 5-10. (in Russian).

5. Lykov A. V. Heat Conduction Theory. Moscow: Vysshaya Shkola, 1967. 600 pp. (in Russian).

6. Słota D. Direct and inverse one-phase Stefan problem solved by the variational iteration method, J. Computers & Mathematics with Applications. 2007.V. 5. Nos. 7-8. Pp. 1139-1146.  
<https://doi.org/10.1016/j.camwa.2006.12.061>

7. Tarkhov D., Vasilyev A. Problems for partial differential equations in the case of the domain with variable borders. Pp. 22-30 (Book chapter). Semi-Empirical Neural Network Modeling and Digital Twins Development. Academic Pr, 2019. Pp. 320 pp.

8. Yu Y., Luo X., Cui H. The solution of two-phase inverse Stefan problem based on a hybrid method with optimization. J. Mathematical Problems in Engineering. 2015. V. 1. Pp. 1-13.  
<https://doi.org/10.1155/2015/319054>

9. Hadži M., Cuo Y. Stability in the Stefan problem with surface tension. J. Communications in Partial Differential Equations. 2010. V. 35. No. 2. Pp. 201-244.  
<https://doi.org/10.1080/03605300903405972>

10. Timoshpolsky V. I., Belyaev N. M., Ryadno A. A. Applied Problems of Metallurgy Thermal Physics. Minsk: Navuka i Tekhnika, 1991. 320 pp. (in Russian).

11. Wells A. J., Hitchen J. R., Parkinson J. R. G. Mushy-layer growth and convection, with application to sea ice, J. The Royal Society. 2019. V. 377. No. 2146. Pp. 377-390.

<https://doi.org/10.1098/rsta.2018.0165>

12. Worster M. G. Natural convection in a mushy layer. Journal of Fluid Mechanics. 1991. V. 224. Pp. 335-339.

<https://doi.org/10.1017/S0022112091001787>

13. Lee D., Alexandrov D., Huang H.-N. Numerical modeling of one-dimensional binary solidification with a mushy layer evolution, J. Numerical Mathematics: Theory, Methods and Applications. 2012. V. 5. No. 2. Pp. 157-185.

<https://doi.org/10.4208/nmtma.2012.m1029>

14. Marangunic P. R., Stampella M. B. Appearance of mushy regions in a symmetrical Stefan problem with vanishing heat capacity. European Journal of Applied Mathematics. 1990. V. 1. No. 2. Pp. 177-187.

<https://doi.org/10.1017/S0956792500000140>

15. Barkovskiy V., Barkovska N., Lopatin O. Probability Theory and Mathematical Statistics. Kyiv: Center of Educational Literature, 2019. 424 pp.

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