UDC 629.78 https://doi.org/10.15407/itm2022.04.014

A. P. ALPATOV[1], A. I. MASLOVA[1,2], A. V. PIROZHENKO[1,2]

PRELIMINARY SELECTION OF REFERENCE ORBIT FOR EARTH REMOTE SENSING SATELLITE

[1] Institute of Technical Mechanics
of the National Academy of Sciences of Ukraine and the State Space Agency of Ukraine,
15 Leshko-Popel St., Dnipro 49005, Ukraine; e-mail: alex.pirozhenko@ukr.net
[2] Earth Observing System Data Analytics, 1906 El Camino Real, Suite 201, Menlo Park, CA 94027, USA

Розглядаються низькі майже кругові орбіти супутників дистанційного зондування Землі (ДЗЗ). Ставиться задача вибору орбіт, що найбільш підходять для конкретної місії супутника. Зокрема, розглядається задача приблизного визначення параметрів орбіт, що дозволяють здійснювати задовільний огляд супутником цільової поверхні Землі. Розглянуто основні побажання розробників систем спостереження відносно умов зйомки поверхні Землі. Узгодженню цих побажань із закономірностями орбітального руху супутників на низьких навколоземних орбітах допомагає, як представляється, використання простих моделей, що описують ці закономірності. При цьому бажана візуалізація смуг огляду на Землі. Компромісним рішенням між побажаннями розробників системи спостереження та закономірностями руху супутників є відбір орбіт, які найбільше відповідають характеристикам супутника та його системі спостереження. Мета статті полягає у викладенні простої моделі та алгоритму, що дозволяють здійснити попередній вибір орбіт супутників ДЗЗ. Пропонована модель спирається на відомі співвідношення і новизна матеріалу полягає у компактному та узагальненому викладенні моделі для попереднього вибору орбіти супутника ДЗЗ. У статті наведено моделі, що дозволяють здійснити: оцінку ширини смуги огляду супутника; вибір кута нахилу орбіти; вибір стійкої форми орбіти; вибір висоти орбіти; вибір періоду орбітального руху. Розглянуто переваги та недоліки сонячно-синхронних орбіт. Побудовано аналітичні вирази, що дозволяють досить просто оцінити відхилення супутника від робочої орбіти під дією аеродинамічного гальмування, оцінити швидкість відновлення параметрів орбіти під дією постійного управляючого трансверсального прискорення і визначити допустимі інтервали часу між включеннями двигунів і інтервали їх роботи. Показано переваги орбіт наземного маршруту, що повторюється. Побудовано найпростішу модель для розрахунку та візуалізації смуг огляду супутником поверхні Землі. Таким чином, у статті запропоновано простий алгоритм попереднього вибору орбіт супутників ДЗЗ на низьких навколоземних орбітах, які забезпечують задовільний огляд цільової поверхні Землі.

Ключові слова: *супутник дистанційного зондування Землі, робоча орбіта, ширина смуги огляду, кут нахилу орбіти, форма орбіти*.

Low near-circular orbits of Earth remote sensing (ERS) satellites are considered. The objective is to select the orbits most suitable for a particular satellite mission. In particular, the problem of an approximate determination of the orbit parameters that allow a satisfactory satellite survey of the target surface of the Earth is considered. The main desires of observation system developers regarding the conditions of the Earth's surface survey are considered. To reconcile these desires with the regularities of satellite motion in low Earth orbits, use may be made of simple models that describe these regularities. In doing so, it is desirable to visualize viewing swaths on the Earth's surface. A compromise between the desires of observation system developers and the satellite motion regularities is the selection of orbits that best meet the characteristics of a particular satellite and its observation system. This article presents a simple model and algorithm that make it possible to preselect ERS satellite orbits. The proposed model is based on familiar relationships, and the novelty of the article lies in a compact and generalized presentation of the model for ERS satellite orbit preselection. The article presents models that make it possible to estimate the satellite swath width and choose the orbit inclination angle, a stable orbit shape, the orbit altitude, and the orbital period. The advantages and disadvantages of solar synchronous orbits are considered. Analytical expressions are constructed to fairly simply estimate the excursion of a satellite from its operational orbit under the action of the aerodynamic drag, estimate the rate of recovery of the orbit parameters under the action of a constant transversal control acceleration, and determine allowable time intervals between engine starts and engine operation intervals. The advantages of repeat ground track orbits are shown. The simplest model for calculating and visualizing satellite viewing swathes of the Earth's surface is constructed. Thus, the article proposes a simple algorithm for the preselection of low Earth orbits for ERS satellites with a satisfactory observation of the target surface of the Earth.

Keywords: *Earth remote sensing satellite, reference orbit, viewing swath width, inclination angle, orbit shape.*

Introduction. The choice of the reference orbit for the Earth remote sensing satellite is a necessary and important mission planning task, the solution of which largely determines the effectiveness of the entire mission. The choice of orbits is a multifaceted and iterative process associated with understanding the goals and capabilities of the mission, and which has no absolute rules [1]. The task of preliminary orbit selection for remote sensing satellites is much simpler and is associated with an approximate determination of the orbit parameters that allow a satisfactory (better is always wanted) survey by the satellite of the Earth's target surface.

In the Western scientific literature, the problems of choosing the orbits for remote sensing satellites were intensively discussed in the first decade of our century [2 – 5]. And at present, interest in this topic remains (see, for example, [6, 7]). In Ukraine, certain issues of choosing the orbit parameters for remote sensing satellites and conducting surveys of the Earth's surface have been considered (see, for example, [8 – 10]), however, a number of important tasks of choosing the orbits for remote sensing satellites remained without discussion. This article aims at filling this gap and proposes a simple algorithm for preliminary orbit selection for remote sensing satellites that are most suitable for the mission under consideration.

Formulation of the problem. Low near-Earth almost circular orbits for satellites have been considered. For definiteness, we consider orbits with an altitude of 500 km to 600 km. It is necessary to choose the orbits for the satellite that best correspond to the technical characteristics of the satellite, including its system for observing the Earth's surface.

We will use the following definitions:

Swath width is the width of the Earth's surface area that falls into the instantaneous frame of the scanner.

Viewing swath width is the width of the area of the Earth's surface that can fall into the instantaneous frame of the scanner, taking into account changes in the angular position of the satellite (rotation in the roll angle).

The wishes of the developers of the sensing system to the characteristics of the orbit are usually the following:

– as close as possible to the shooting surface;

– as often as possible to be able to shoot an arbitrary or given area of the Earth's surface;

– as soon as possible to ensure sufficient coverage of viewing swaths (overlapping viewing swaths increases the speed of creating a complete picture of a given area, the width of which is greater than the swath width);

– the constancy of the distance and speed of the satellite relative to the surface to be photographed;

– and etc.

To match these wishes with reality, simple arguments about the patterns of orbital motion and, preferably, images of viewing swaths on the Earth's surface, will help. A compromise solution is to select the orbits that best match the characteristics of the satellite and its observation system. The purpose of the article is to present a simple model and reasoning that allows a preliminary choice of orbits.

Note that the proposed model is based on well-known relations (see, for example, [1, 5]) and the novelty of the material lies in a compact and generalized presentation of the model for the preliminary selection of the Earth remote sensing satellite orbit.

Model for Preliminary Selection of Earth Remote Sensing Satellite Orbit. Viewing swath width estimation.

We find the viewing swath width assuming the sphericity of the Earth's surface Fig. 1.

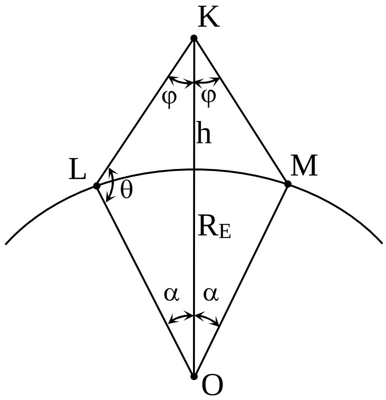

Fig. 1

From triangles, $\sin ¯ = \sin ¨ \frac{R_0}{R_E}$, where $R_E, R_0 = R_E + h$ - the radius of the Earth and the orbit, respectively, h is the altitude of the orbit, φ is the deviation the observation system axis (the rotation angle of the satellite relative to the local vertical, the roll angle). Since $¯ \geq 90^{o}$, then $¯ = ® - \arcsin\left(\sin ¨ \frac{R_0}{R_E}\right)$. Thus,

$$~ = \arcsin\left(\sin ¨ \frac{R_0}{R_E}\right) - ¨ \ . \qquad (1)$$

The deviation of the observation system axis from the sub-satellite point on the Earth's surface in meters can be estimated by the formula $\Delta_{ot} = R_E ~$.

For the orbits under consideration $(500 \leq h \leq 600)$ km, the viewing swath width increases approximately linearly with increasing satellite altitude. For example, for $\varphi = 45^{o}$ the value Δ_{ot} increases from ≈522 km at h =500 km to ≈632 km at h =600 km, i.e. approximately 20 %. The axis of the observation system reaches the horizon for h =500 km at $\varphi \approx 68^{o}$ (Δ_{ot} ≈2445 km).

The choice of the inclination angle. The orbital inclination angle i determines the orbital plane precession rate (the rate of the longitude change of the ascending node Ω) for low near-earth orbits (LEO). In the first approximation, the secular motion Ω for almost circular orbits can be described by the equation [11]

$$\dot{\Omega} = -\varepsilon \sqrt{\frac{\mu}{R_0^3}} \cos i \ , \qquad (2)$$

where $\varepsilon = -\frac{3}{2} C_{20} \frac{R_z^2}{R_0^2}$; $C_{20} \approx -1.0826 \cdot 10^{-3}$ is the coefficient at the second zonal harmonic of the Earth's gravitational field expansion; R_z is the average equatorial radius of the Earth; R_0 is the average radius of the orbit; μ is the gravitational constant of the Earth.

The most popular orbits for remote sensing satellites are solar synchronous orbits, the orbits for which the secular motion Ω is equal to the average motion of the Sun along the equator. Since the Earth makes a complete revolution relative to the Sun in one tropical year, then the average angular velocity of the Sun along the equator relative to the inertial equatorial coordinate system equals [5] .

$$\dot{a}_C = \frac{360 \text{ deg}}{365.2421897 \; day} = 0.98564736 \frac{\text{deg}}{day} = 1.9910638534\text{E-}07 \frac{1}{\text{c}}. \qquad (3)$$

Equating (2) and (3) we obtain a relation connecting the orbit parameters for solar synchronous orbits.

$$\dot{\lambda}_C = -\varepsilon \sqrt{\frac{\mu}{R_0^3}} \cos i \; . \qquad (4)$$

It can be seen from (4) that small changes in the average radius of the orbit will lead to small changes in the inclination angle. So, when the average orbit altitude changes from 500 km to 600 km, the inclination angle according to (4) changes from ≈ 97.375° up to ≈97.76°. That is, low near-earth solar synchronous orbits (SSO) are subpolar orbits.

The advantages of the SSO are obvious: constant good illumination of the Earth's surface to be photographed for satellites with optical observation systems (day-night orbit), and constant illumination of satellites with radar (dawn-dusk orbit).

At the same time, the proximity of the SSO to polar orbits leads to the fact that satellites fly over territories that are usually of little interest for observation, the circumpolar regions, for a significant part of the time. Thus, the main part of Europe lies at latitudes up to 60°, and to survey the territory of Ukraine, it is enough to be able to view latitudes up to 52.5°.

Let us determine the orbit inclination i , which will allow the remote sensing satellite to survey the regions of the Earth's surface with a latitude less than a certain limiting value $\delta_{\max}$. In the first approximation, we can assume that the sub-satellite point reaches its maximum latitude at the latitude argument $u = 90^{\circ}$ (this, in particular, results from the simple model given below). At this point (at $u = 90^{\circ}$), the latitude of the sub-satellite point will be equal to the orbit inclination i , and the satellite moves along the parallel. Then the maximum latitude $\delta_{\max}$, which can be observed by the satellite, is determined as follows: $\delta_{\max} = i + \delta_S$, where δ_S is the addition to the latitude, which depends on the maximum possible deviation of the observation instrument axis from the local vertical. It is easy to see that $\delta_S = \alpha$, where α is determined by formula (1). Thus, for a satellite orientation deviation of $\varphi = 45^{\circ}$, δ_S will change almost linearly from ≈4.7° to ≈5.7° as the orbit altitude changes from 500 km to 600 km. Therefore, in this case, to survey the main part of Europe, the orbital inclination should be no less than ≈55°, and for Ukraine ≈47°.

Note that the benefits of using the SSO, associated with the stability of the satellite illumination by the Sun and the underlying surface of the Earth, make these orbits the most attractive for low-cost commercial remote sensing satellites.

The choice of the orbit shape. The desire of the developers of the sensing system for the constancy of the distance and speed of the satellite relative to the surfaces being photographed is not feasible. The compression of the Earth from the poles causes the difference between the mean equatorial radius of the Earth and the polar one by ≈21 km. Therefore, even for a circular Keplerian SSO – an orbit with a constant radius, it is impossible to achieve a constancy of the distance between the satellite and the sub-satellite point.

There are so-called orbits of minimum altitude change (OMAC) [12, 13]. These SSOs have two apogees – above the equator, and two perigees - above the poles. This shape of the orbit reduces the changes in the orbit altitude above the common Earth ellipsoid to approximately 18 km. However, for subpolar orbits, OMAC are unstable, and their maintenance requires quite frequent switching on of corrective engines. The long-term evolution of uncontrolled motion under the influence of zonal harmonics [5] can significantly increase the altitude changes of orbits. Long-term changes in the orbit altitude lead to corresponding changes in other characteristics of the orbit, including its period.

Therefore, the optimal solution for SSO lies in the choice of stable orbits - orbits for which the natural drift of the orbit shape under the influence of the Earth's gravitational field is minimized by careful selection of orbit parameters. For such orbits, for a long time, the altitude and speed of the satellite remain almost constant at the same point in the orbit on each revolution. Such orbits require minimal control input to maintain their shape.

In the first approximation for the SSO, the average orbital elements of the frozen orbits are determined by the equalities [5]

$$\cos\omega = 0, \qquad e = -\frac{1}{2}\frac{C_3}{C_2}\frac{R_z}{a}\sin i, \tag{5}$$

where e is the eccentricity of the orbit; $C_{30} \approx 2.5324 \cdot 10^{-6}$ is the coefficient at the third zonal harmonic of the geopotential expansion; a is the semi-major axis of the orbit; ω is the argument of the orbit perigee.

It is easy to see from (5) that the average elements have the values $\omega = 90^{o}$, 270^{o}, and for LEO it's $e \approx 0.001$.

For the orbital parameters describing the orbit deviation from the circular comparison orbit, in the first approximation, the shape of the stable orbit is determined by the equalities [14]

$$\begin{aligned} \overline{A}_0 \sin\overline{\alpha}_0 &= -\frac{C}{G}, \\ \overline{A}_0 \cos\overline{\alpha}_0 &= \frac{d}{3}, \end{aligned} \tag{6}$$

where $\overline{A}_0, \overline{\alpha}_0$ are the initial average amplitude and phase shift (argument of the apogee) of the orbit's natural oscillations relative to the circular orbit of radius R_0; $G = 5d - 2\varepsilon$; $C = \frac{3}{2}\varepsilon_3 \sin\bar{i}\left(\frac{5}{4}\sin^2\bar{i} - 1\right)$; $d = \frac{\varepsilon}{2}\sin^2\bar{i}$; $\varepsilon_3 = \frac{C_{30}R_z^3}{R_0^3}$.

From (6) it is easy to obtain $\overline{A}_0 \approx 0.001$, $\overline{\alpha}_0 \approx 282^{o}$.

Note that the formulas (5), (6) are given for the average elements, which are determined in accordance with the accepted models for describing the satellite motion under the action of the geopotential zonal harmonics (i.e., each model has its own average elements). In addition, the formulas are given only taking into account the second and third zonal harmonics. To select the orbit parameters taking into account the more accurate effect of the geopotential on the satellite motion, rather complex iterative calculations are required.

We also note that the second zonal harmonic of the geopotential significantly affects the shape of the satellite trajectory for almost circular ($e \approx 0.001$) LEOs, and the orbit shape differs significantly from an ellipse. Thus, Fig. 2 (curve 1) shows the shape of a frozen SSO for h ≈510 km. For better clarity, the figure shows changes in the trajectory about a circle of radius $R_{cE} + 500$ km, where R_{cE} is the average radius of the Earth. On the axis $O\eta$, lying in the plane of the orbit and directed to the ascending node, the value $(h_t - 500\text{ km})\cos u$ is plotted, where h_t is the altitude of the orbit above the average radius of the Earth. The value $(h_t - 500\text{ km})\sin u$ is plotted along the axis $O\xi$, lying in the plane of the orbit and directed perpendicularly to $O\eta$ towards the north pole of the Earth. The calculations have been carried out taking into account 100×100 harmonics of the Earth's gravitational field decomposition. As can be seen from the figure, the shape of the orbit - this "tomato", slightly resembles an ellipse (curve 2 in Fig. 2) of an unperturbed Keplerian orbit (curve 2 is constructed similarly to curve 1, but only the central field of the Earth acts on the satellite). It seems that it is more convenient to describe such a trajectory by its deviations from a circle than by rotation and deformations of an ellipse.

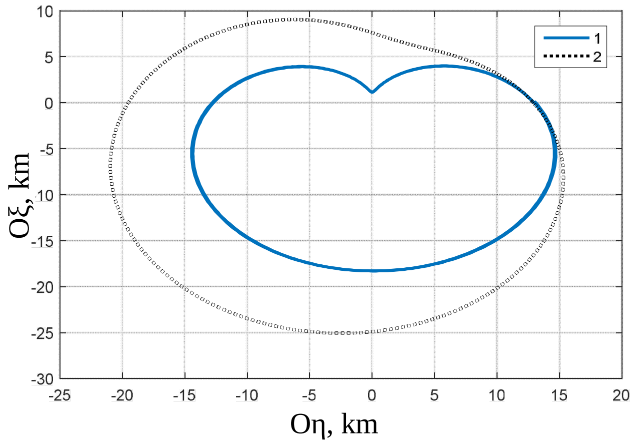

Fig. 2

Choice of orbit altitude. The altitude of the LEO is in many respects the determining value for the aerodynamic drag force of the satellite, which is the main perturbation leading to the orbit degradation.

The aerodynamic acceleration of the satellite can be written as

$$\vec{F}_{aer} = -B\rho V^2 \vec{e}_v \,, \tag{7}$$

where $B = cS/2m$ is the ballistic coefficient of the satellite; c is the drag coefficient of the satellite, S is the area of aerodynamic drag (midship section) of the satellite, $\vec{V}$ is the velocity of the satellite relative to the oncoming flow, $\vec{V} = \vec{V}_o + \vec{V}_w$, $\vec{V}_o$ is the velocity vector of the satellite's orbital motion, $\vec{V}_w$ is the velocity vector of the rotating atmosphere, $V = |\vec{V}|$, $\vec{e}_v = \vec{V}/V$, ρ is the density of the atmosphere.

To estimate the aerodynamic acceleration of the satellite on the SSO, one can neglect the wind speed, and taking into account the almost circular shape of the orbit, consider the satellite speed equal to its speed in unperturbed motion

$$\vec{V} \approx \vec{V}_o \approx \sqrt{\frac{\mu}{R_0}} \vec{e}_\tau,$$

where $\vec{e}_\tau$ is the unit vector of the tangent to the orbit and directed to the satellite motion. The drag coefficient of the satellite also depends on the shape of the satellite's surface, but in many cases for the considered altitudes it can be taken equal to $c = 2.1$.

To compensate for aerodynamic braking, low-thrust engines, usually electric propulsion engines, are used. To match the capabilities of the satellite orbital motion control system with the aerodynamic braking acting on the satellite at a given altitude, it is necessary to do a fairly large amount of work:

– estimate the average and maximum possible density of the atmosphere during the planned active operation of the satellite;

– determine the permissible deviations of the satellite from the reference orbit;

– determine the allowable time intervals between switching on the engines and the intervals of their operation.

Sufficient accuracy for estimating the displacement of the satellite relative to the reference orbit when the satellite is subjected to transversal acceleration in almost circular orbits is given by the inhomogeneous Clohessy-Wiltshire equations [5]

$$\begin{aligned} x'' - 2y' - 3x &= 0, \\ y'' + 2x' + \tilde{T} &= 0, \end{aligned} \tag{8}$$

where x, y are the displacements of the satellite relative to its position in the reference orbit along the radius vector and along $\vec{e}_\tau$ respectively; the prime denotes the derivative with respect to the dimensionless time $\tilde{u}$, $\dot{\tilde{u}} = \sqrt{\frac{\mu}{R_0^3}}$; $\tilde{T} = \frac{R_0^3}{\mu} T$.

With the constant $\tilde{T}$ the equations are integrated analytically, and their general solution has the form

$$\begin{aligned} x &= c_2 \sin\tilde{u} + c_3 \cos\tilde{u} + 2c_1 - 2\tilde{T}\tilde{u}, \\ x' &= c_2 \cos\tilde{u} - c_3 \sin\tilde{u} - 2\tilde{T}, \\ y &= 2c_2 \cos\tilde{u} - 2c_3 \sin\tilde{u} - 3c_1\tilde{u} + c_4 + 1.5\tilde{T}\tilde{u}^2, \\ y' &= -2c_2 \sin\tilde{u} - 2c_3 \cos\tilde{u} - 3c_1 + 3\tilde{T}\tilde{u}, \end{aligned} \tag{9}$$

where $c_1 - c_4$ are the integration constants determined by the initial conditions.

Solutions (9) make it possible to fairly simply estimate the deviations of the satellite from the reference orbit under the action of aerodynamic braking, estimate the recovery rate of the orbit parameters under the action of a constant transversal control acceleration, and determine the allowable time intervals between engine starts and intervals of their operation.

The altitude of the orbit is decisive not only for the amount of aerodynamic drag, but also for an important characteristic of the reference orbit - the period of orbital motion.

The choice of the motion period. The nodal (draconian) period is the period of the return of the satellite to the ascending (descending) node, i.e. the period of time between two successive crossings of the equator in the same direction (for example, from south to north).

In the first approximation, taking into account the second zonal harmonic, it can be estimated by the formula [12]

$$P_{\Omega} = P_K\left[1 - 0.5£\left(3 - 3.5\sin^2 i\right)\right], \tag{10}$$

where $P_K = 2\pi\sqrt{\frac{R_0^3}{\mu}}$ is the period of the orbital motion in the unperturbed Keplerian motion; the remaining designations are the same as in formula (2).

For SSO, the nodal period increases from approximately 94 minutes 57 seconds for h =500 km to 96 minutes 34 seconds for h =600 km.

The period of the Earth's rotation relative to the ascending (descending) node is called the nodal period of Greenwich [5]. This Greenwich period on average, as it is easy to understand, can be estimated by the expression

$$P_G = \frac{2\pi}{\omega_3 - \dot{\Omega}},$$

where $\omega_3, \dot{\Omega}$ are the average angular velocities of the Earth's rotation and the longitudes of the ascending node of the orbit in the equatorial ISC, respectively, $\mu_3 = 2® / T_3 \approx 7.292115853 \cdot 10^{-5} 1/c$, where T_3 are the sidereal days.

The earth rotates relative to the orbit plane with an angular velocity $\omega_3 - \dot{\Omega}$. During the period of revolution of the satellite, the Earth will rotate relative to the orbit plane by an angle $P_{\Omega}(\omega_3 - \dot{\Omega})$. For solar synchronous orbits ($\dot{\Omega} = \dot{\lambda}_c$) and altitudes of 500–600 km, this angle lies in the range of 0.412–0.421 radian (23.60–24.10 degree). At the equator, this corresponds to the displacement of the satellite track by $R_3 P_{\Omega}(\omega_3 - \dot{\Omega})$, or in numbers ≈2630 ... 2687.5 km. At latitude δ the shift of the sub-satellite point per revolution of the satellite can be estimated by the formula $R_{c3} \cos\delta P_{\Omega}(\omega_3 - \dot{\Omega})$, and, for example, for δ =48º (Ukraine) is from 1760 km to 1798 km. Thus, for example, it follows that if the axis of the observation system can be deflected by an angle $\varphi \approx 56^{o}$ ($\Delta_{ot} = R_z \alpha$ =900 km) in both directions from the motion direction, the satellite will be able to observe an arbitrary point on the Earth's surface at the latitudes of Ukraine daily.

Typically, the satellite's viewing swath width is quite narrow, and the satellite can only observe a fairly narrow swath between the projections of the satellite's paths on the Earth's surface. This is primarily due to the desire for the quality of the obtained observational information. In this case, the task becomes to choose the period of satellite motion that provides the best conditions for surveying a given area. The optimal solution to this problem, apparently, lies on the set of orbits of a repeating ground route [5].

To construct an orbit for a repeating ground route, it is necessary to choose such parameters of the orbit that would satisfy the relation

$$nP_G = mP_\Omega \quad \text{or} \quad P_G / P_\Omega = m/n = N\,\frac{m_1}{n_1},$$

where n, m, N, m_1, n_1 are integers; N is the number of full satellite revolutions in orbit per day for SSO; $\frac{m_1}{n_1}$ is the fractional part of the ratios for the periods P_G / P_Ω.

For stable SSOs, the satellite will make revolutions in orbit in n_1 days and return to its original point relative to the Earth - the orbit will “close”. This makes it possible to re-survey the observation area under practically the same conditions as the previous one: the distance and speed of the satellite relative to the surface being photographed are almost the same, and the angle of the sun to illuminate the surface is almost the same.

In addition to the stability of the time intervals between observations of a given area, the periodicity of the process can significantly reduce the cost of ground-based maintenance of the remote sensing system.

Figure 3, as an example, shows the changes in the ratio of the Greenwich nodal period to the nodal period depending on the altitude for different orbital inclinations. On the ordinate axis, the ratio of periods providing Repeat-groundtrack orbits (RGT) are from 4 to 10 days.

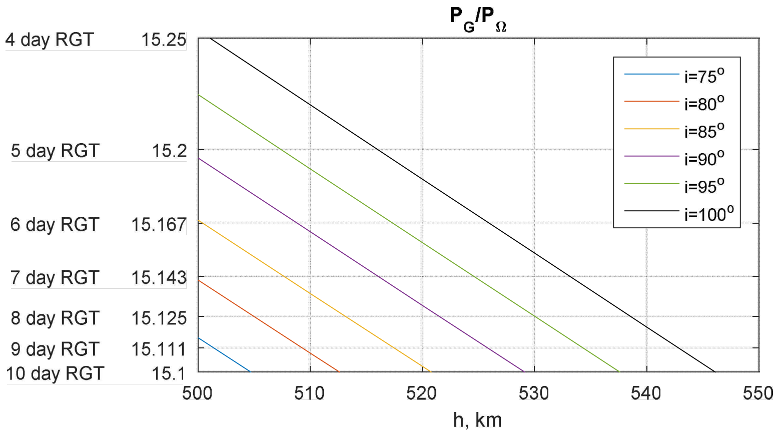

Fig. 3

Of course, the choice of the satellite motion period should provide viewing swath coverage of the given observation area. This choice is facilitated by computer calculations and their visualization using the simple model is proposed below.

Model for calculation of viewing swaths and their visualization. The simple model is built under the following assumptions:

– the satellite’s orbit is circular;

– the satellite moves uniformly in orbit with an angular velocity equal to $\omega_{or} = 2\pi / P_\Omega$;

– orbital inclination i is constant;

– changes in the longitude of the ascending node Ω are determined by formula (2).

Let us introduce a right coordinate system $OXYZ$ associated with the Earth with the origin at the center of mass of the Earth O and associated with some point on the equator (for example, with the Greenwich meridian): the axis OX lies in the plane of the equator and is directed to a fixed point on the equator; the axis OZ is directed along the axis of the Earth's rotation towards the north pole, the axis OY complements the system to the right. The system rotates relative to the equatorial inertial coordinate system together with the Earth with an angular velocity ω_3.

Let us introduce a right orbital coordinate system $Oxyz$ with the origin at the Earth's center of mass O: the axis Ox is directed along the radius vector of the orbit $\vec{R}$, the axis Oy is in the plane of the instantaneous orbit in the direction of the satellite's motion; the axis Oz is along the binormal to the orbit.

The orientation $Oxyz$ in $OXYZ$ will be described by Euler angles i, Ω_b, u – inclination, longitude of the ascending node and latitude argument, respectively. The angle u, due to the assumptions made, is defined as $u = \omega_{or} t$, where t is the time. The change rate Ω_b is determined by the rate of the Earth's rotation and the orbital precession rate $\dot{\Omega}_b = -(\omega_3 - \dot{\Omega})$.

Thus, the orientation of $Oxyz$ in $OXYZ$ is defined by:

$$i = \text{const}, \;\; u = \mu_{or} t, \;\; \Omega_b = \Omega_{b0} - (\mu_3 - \dot{\Omega})t.$$

Where Ω_{b0} is the deviation along the equator of the first ascending node of the trajectory.

Let us determine the longitude and latitude of a point on the Earth's surface at $OXYZ$, when the radius vector $\vec{R}_t$ of the point is given at $Oxyz$. Let's denote the longitude of the point as λ, $0^\circ \le \lambda \le 360^\circ$, latitude – δ, $-90^\circ \le \delta \le 90^\circ$.

The coordinates of a point in $OXYZ$ through the coordinates of a point in $Oxyz$ are determined through the matrix of direction cosines

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \cos u \cos\Omega_b - \sin u \sin\Omega_b \cos i, & -\sin u \cos\Omega_b - \cos u \sin\Omega_b \cos i, & \sin i \sin\Omega_b \\ \cos u \sin\Omega_b + \sin u \cos\Omega_b \cos i, & -\sin u \sin\Omega_b + \cos u \cos\Omega_b \cos i, & -\sin i \cos\Omega_b \\ \sin u \sin i, & \cos u \sin i, & \cos i \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

Knowing the coordinates of the point in $OXYZ$ we find the spherical angles

$$\theta = \arccos(Z / R_{ct}) = \text{arctg}\left(\frac{\sqrt{X^2 + Y^2}}{Z} \right),$$

$$\lambda = \text{arctg}\left(\frac{Y}{X}\right), \quad \cos\lambda = \frac{X}{R_{ct}\sin\theta}, \quad \sin\lambda = \frac{Y}{R_{ct}\sin\theta},$$

where R_{ct} is the radius of the Earth at the desired point, $\delta = \pi/2 - \theta$.

For the sub-satellite point, the coordinates in $Oxyz$ are $(R_{ct}, 0, 0)$.

Let us find the coordinates of the points of the viewing swath boundaries. From Fig. 1, the coordinate of the point L is determined by the vector $\overrightarrow{OK} + \overrightarrow{KL}$, where $\overrightarrow{OK} = \vec{R} = R_0 \vec{e}_x$ is the current radius vector of the satellite, φ is the roll deviation of the observation system axis from the local vertical. Having determined $\alpha = \arcsin\left(\sin\varphi \frac{R}{R_{ct}}\right) - \varphi$, we find the length KL, $|KL| = \sin\alpha \frac{R_{ct}}{\sin\varphi}$. Then the coordinates of the point L in $Oxyz$ are $(R - |KL|\cos\varphi, 0, \pm|KL|\sin\varphi)$, where the "–" sign corresponds to the satellite motion from the figure to the observer, and the + sign corresponds to the reverse motion. The coordinates of the point M are determined similarly.

The proposed model allows you to simply and quickly calculate and demonstrate the coverage of the Earth's surface with viewing swaths for various types of observation systems, changes in the orientation of the satellite and the parameters of its orbit.

Conclusions. A simple model of satellite motion in LEO is proposed, which takes into account the main regularities of this motion. The features of the choice of the inclination angle, shape, altitude and period of the orbit have been considered. The simple model for calculating the coverage of the Earth's surface with viewing swaths and their visualization has been proposed. In general, a simple algorithm is proposed for the preliminary orbit selection for remote sensing satellites in LEO, which provide a satisfactory view of the target Earth's surface.

1. *Larson W. J., Wertz J. R.* Space Mission Analysis and Design. Third Edition. 1999. 9786 p.
2. *Vadali R., Kyle T. A.* Satellite Orbit Design and Maintenance for Terrestrial Coverag Prasenjit Sengupta Srinivas. Journal of spacecraft and rockets. 2010. Vol. 47. № 1. P. 177–187. https://doi.org/10.2514/1.44120
3. *Aorpimai M. Palmer P. L.* Repeat-Groundtrack Orbit Acquisition and Maintenance for Earth-Observation Satellites. Journal of Guidance, Control, and Dynamics. 2007. Vol. 30, № 3. P. 786–793. https://doi.org/10.2514/1.23413
4. *Abramson W. R., Carter D., Kolitz S., McConnell J., Ricard M., Sanders C.* The Design and Implementation of Draper's Earth Phenomena Observing System. AIAA Space 2001 Conference & Exposition. 2001. https://doi.org/10.2514/6.2001-4565
5. *Vallado D. A.* Fundamentals of Astrodynamics and Applications. Fourth Edition. Space Technology Library. 2013. 1106 p.
6. *Pu M., Wang J., Zhang D., Jia Q., Shao X.* Optimal Small Satellite Orbit Design Based on Robust Multi-Objective Optimization Method. Aerospace Science and Technology. 2017. Vol. 70. P. 339–350. https://doi.org/10.1016/j.ast.2017.08.016
7. *Shilin Z.* Satellite Orbit Design and Analysis Based on STK. Journal of Physics: Conference Series. 2021. Vol. 2228. № 1. id.012037, 5 pp. https://doi.org/10.1088/1742-6596/2228/1/012037
8. *Ivanova V. I., Sheptun A. D.* Minimization of deviation from local sun time of sun synchronous orbit ascending node considering injection accuracy. Aerospace technic and technology. 2015. № 2 (119). C. 52–56.
9. *Borshcheva G. A., Masley V. N., Shovkoplyas Yu. A., Yarmolchuk E. D.* Structure and main characteristics of the space system "Sich-2". Space technology. Missile armaments. 2015. № 2. C. 16–24.
10. *Lyashenko, V.V., Yatsun, L.V., Yarmolchuk, Ye.D.* Satellite "Sich-2M" survey planning. Space science and technology. 2020. № 1(122). C. 30–36. https://doi.org/10.15407/knit2020.01.030
11. *Pirozhenko A. V., Maslova A. I., Vasilyev V. V.* About the influence of second zonal harmonic on the motion of satellite in almost circular orbits. Space science and technology. 2019. Vol. 25, № 2. C. 3–14. https://doi.org/10.15407/knit2019.02.003
12. *Maslova A. I., Pirozhenko A. V., Vasyliev V. V.* Minimum altitude variation orbits. Analysis of characteristics and stability. Technical mechanics. 2021. № 4. P. 44–55. https://doi.org/10.15407/itm2021.04.044
13. *Artyushenko V. M., Vinogradov D. Y.* Analysis of the properties of orbit minimum elevation changes. Information Technology Bulletin. 2017. № 4 (14). C. 3–15
14. *Pirozhenko A. V., Maslova A. I., Vasyliev V. V.* Analytical model of satellite motion in almost circular orbits under the influence of zonal harmonics of geopotential. Space Science and Technology. 2022. 28, № 4 (137). P. 18–30. https://doi.org/10.15407/knit2022.04.018

Received on November 15, 2022,
in final form on February 10, 2023