УДК 629.78 https://doi.org/10.15407/itm2023.03.034

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MATHEMATICAL MODEL FOR DETERMINING THE DESIGN PARAMETERS OF THE AERODYNAMIC ELEMENTS OF A DEORBIT SYSTEM

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Мета статті – розробка моделі для вибору проєктних параметрів аеродинамічних елементів систем відведення з орбіти. Для вирішення проблеми засмічення навколоземного простору пропонується відводити з орбіти відпрацьовані космічні об'єкти. Найбільш засміченим є регіон низьких навколоземних орбіт. Однією з перспективних систем відведення космічних об'єктів з низьких навколоземних орбіт є аеродинамічні системи відведення. Такі системи досить надійні і дешеві, але вони мають і недоліки, а саме чутливість до впливу факторів космічного простору. Виконано декомпозицію аеродинамічних систем. В результаті декомпозиції визначено ієрархічну структуру системи, яка має наступні рівні: рівень підсистем, рівень елементів, рівень параметрів. Проведено аналіз матеріалів для виготовлення конструктивних елементів аеродинамічного елемента системи. Сформовано комплекс проєктних параметрів аеродинамічних систем. Даний комплекс використано при розробці математичної моделі вибору параметрів аеродинамічного елемента систем відведення різних класів, а саме: моноблочних, каркасно-надувних, систем, які формуються шляхом трансформації конструкції космічного об'єкта в аеродинамічну систему, висувних систем. В моделі визначення товщини матеріалу враховано наступні фактори: вплив космічного вакууму та атомарного кисню на матеріал оболонки, вплив надлишкового тиску на оболонку. Також в моделі враховано похибки: у визначенні балістичного коефіцієнту аеродинамічної системи з космічним об'єктом, що відводиться, у визначенні індексу сонячної активності та у визначенні концентрації атомарного кисню. Дана математична модель вибору параметрів аеродинамічної системи дозволяє будувати номограми для визначення параметрів систем відведення космічних об'єктів різних класів за масою та орбіт дислокації.

Ключові слова: *космічний об'єкт, система відведення з орбіти, аеродинамічний елемент, комплекс проєктних параметрів, математична модель.*

The goal of this paper is to develop a mathematical model for choosing the design parameters of deorbit systems' aerodynamic elements. To solve the problem of near-Earth space debris, it is proposed to deorbit used space objects. Low-Earth orbits are most clogged. Aerodynamic systems are among the most promising systems for space debris removal from low-Earth orbits. They are quite reliable and cheap, but they are sensitive to exposure to space factors. In this paper, aerodynamic systems are decomposed to identify their hierarchic structure, which has the following levels: a subsystem level, an element level, and a parameter level. Materials for the structural components of an aerodynamic element are analyzed. A set of design parameters for aerodynamic systems is formed and used in the development of a mathematical model for choosing the parameters of an aerodynamic element for deorbit systems of various classes: monoblock ones, frame inflatable ones, ones formed by transforming the structure of a space object into an aerodynamic system, and telescopic ones. The material thickness determination model accounts for shell exposure to the space vacuum, atomic oxygen, and excess pressure. It also accounts for errors in determining the ballistic coefficient of an aerodynamic system with a space debris object to be deorbited, the solar activity index, and the atomic oxygen density. The mathematical model for aerodynamic system parameter choice allows one to construct nomograms for determining the parameters of deorbit systems for space debris objects of various classes from their mass and orbit parameters.

Keywords: *space object, deorbit system, aerodynamic element, set of design parameters, mathematical model.*

Introduction. The state of near space today is in a critical condition due to its pollution. Humanity faces the problem of keeping it in a state suitable for practical use. This requires some effort to develop modern technologies to prevent the growth of space debris fragments in outer space by deorbiting space objects that have expired [1]. Aerodynamic systems are one of the most promising technologies for the deorbiting of spent space objects. Passive means, which include this technology, practically do not require fuel and on-board energy for their operation. A characteristic feature of these systems is the creation of a braking force by increasing the ballistic coefficient of the spacecraft. Therefore, such systems are quite reliable and cheap.

Earlier, we classified these systems [2] and proposed a methodology for their designing [3]. Using the classifier and methodology, this paper proposes to develop a model for aerodynamic element design parameters selecting of deorbiting systems.

As you know, the structure and functioning are the most important properties of the system. The functioning of the system is clearly determined by the structure. According to [4]: "structure (lat. structura – structure, connection) is a strong, relatively stable connection (relationship) and interaction of elements, parties, parts of an object, phenomenon, process as a whole." An ordered set of elements that make up a technical system is grouped by systems of lower levels of complexity. The hierarchical structure of the aerodynamic deorbiting systems (ADS) and the composition of the corresponding project parameters are shown in fig. 1.

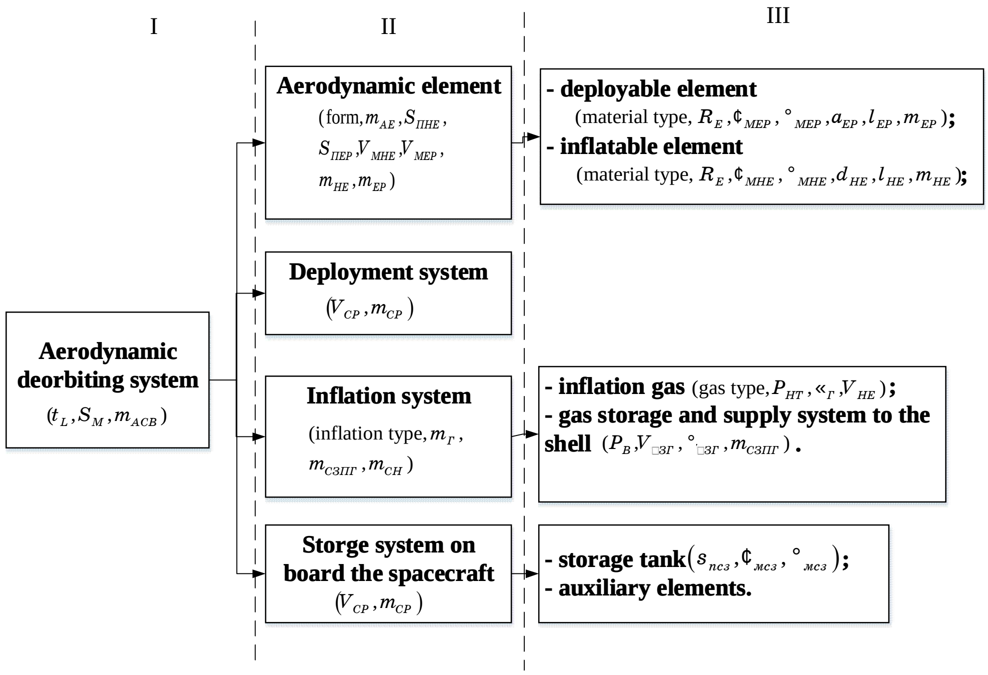


Fig. 1 – Hierarchical structure of ADS

In fig. 1 is shown: I, II, III – levels of systems, subsystems and elements, respectively; t_L is the space object orbital lifetime; S_M is the cross-section area of the aerodynamic element; m_{ACB} is the ADS mass; m_{AE} is the mass of the aerodynamic element; $S_{ПНЕ}$ is the total surface area of inflatable elements; $S_{ПЕР}$ is the cross-section area of deployable elements; V_{MHE} is the volume of inflatable elements; V_{MEP} is the volume of deployable elements; m_{HE} is the mass of inflatable elements; m_{EP} is the mass of deployable elements; V_{CP} is the volume of deployment system; m_{CP} is the mass of deployment system; $m_Г$ is the mass of gas for inflation; $m_{СЗПГ}$ is the mass of gas storage and supply system to the shell; m_{CH} is the mass of inflation system; R_E is the volume coefficient of deorbited mass under the action of atomic oxygen in near-Earth space; ¢_{MEP} is the material thickness of deployable elements; °_{MEP} is the material density of deployable elements; a_{EP} is the width of a deployable element; l_{EP} is the length of a deployable element;

m_{EP} is the mass of deployable elements; $¢_{МНЕ}$ is the material thickness of inflatable elements; $°_{МНЕ}$ is the material density of inflatable elements; $d_{НЕ}$ is the diameter of an inflatable element; $l_{НЕ}$ is the length of an inflatable element; $P_{НТ}$ is the excess pressure in an inflatable element; $«_{Г}$ is the molecular mass of gas for inflation; $V_{НЕ}$ is the volume of inflatable elements; $P_{В}$ is the internal pressure in the gas storage due to inflation; $V_{□ЗГ}$ is the volume of the gas storage tank; $°_{□ЗГ}$ is the density of the gas storage tank material; $m_{СЗПГ}$ is the mass of the gas storage and supply system to the shell; $S_{ПСЗ}$ is the surface area of the storage system; $¢_{МСЗ}$ is the material thickness of the storage system; $°_{МСЗ}$ is the density of the material of the storage system.

The following initial data are used to calculate the parameters of the deorbit system: the lifetime, the mass of the deorbited space object and its cross-sectional area, the cross-sectional area of the aerodynamic element. The latter is calculated according to the following model [5]:

$$S_M = \frac{2m_{КА}\sqrt{\dfrac{a}{\mu}}\cdot X(e,z)}{t_L 3\rho_{pe} C_X} + S_{МКО}, \tag{1}$$

$$X(e,z) = \frac{3\cdot e\cdot \exp(z)}{4I_0(z)+8eI_1(z)}\left\{1+\frac{7e}{6}+\frac{5e^2}{16}+\frac{1}{2z}\cdot\left(1+\frac{11e}{12}+\frac{3}{4z}+\frac{3}{4z^2}\right)\right\}, \tag{2}$$

where C_X is the coefficient of aerodynamic resistance; ρ_{pe} is the density of the atmosphere at the perigee of the orbit; $I_k(z)$ – Bessel functions of order k=0 and 1 and argument $z = ae/H_{\rho,pe}$; e – orbital eccentricity; μ – gravitational parameter of the Earth; $m_{КА}$ – spacecraft mass; a – the semi-major axis of the orbit; $H_{\rho,pe}$ – height of the dense atmosphere.

In general, the mass of the aerodynamic element is determined using the expressions:

$$m_{АЭ} = S_{ПНЭ}\delta_{МНЭ}\rho_{МНЭ} + S_{ПРЭ}\delta_{МРЭ}\rho_{МРЭ}, \tag{3}$$

where $m_{АЭ}$ is mass of AE; $S_{ПНЭ}$ is the surface area of inflatable elements; $\delta_{МНЭ}$ is the material thickness of inflatable element; $\rho_{МНЭ}$ is the material density of inflatable elements; $S_{ПРЭ}$ is the surface area of deployable elements; $\delta_{МРЭ}$ is the material thickness of deployable elements; $\rho_{МРЭ}$ is the material density of deployable elements.

These parameters are calculated according to the following algorithm and corresponding mathematical models.

The analysis of the ADS design schemes, the descriptions of which are given in [2], showed that polymer film materials are used for the manufacture of aerodynamic elements, the characteristics of which are given in Table 1.

Table 1 – Characteristics of polymer film materials

Material	Volumetric coefficient of mass loss $R_e, \text{cm}^3/\text{atom}$	Density $\rho, \text{kg}/\text{m}^3$	Modulus of elasticity σ_{np}, Pa
Mylar	$3,01\cdot10^{-24}$ [6]	1390 [7]	$3,38\cdot10^{9}$ [7]
Upilex-S	$9,22\cdot10^{-25}$ [6]	1470 [10]	$9,1\cdot10^{10}$ [10]
Kapton-H	$3\cdot10^{-24}$ [6]	1420 [8]	$2,76\cdot10^{10}$ [8]
PTFE	$1,42\cdot10^{-25}$ [6]	2150 [6]	$1,75\cdot10^{10}$ [9]
Kapton-Al_2O_3	$2,5\cdot10^{-26}$ [11]	1390 [8]	$3,38\cdot10^{9}$ [8]
Kapton-FN	$5\cdot10^{-26}$ [11]	1530 [12]	$2,48\cdot10^{10}$ [12]

The design value of the material thickness of inflatable and deployable elements is determined by the formula:

$$\delta_{MHE} = \delta_{MPE} = \delta_0 + \left(\delta_C + \delta_A\right)\cdot\left(1+\bar{\delta}\right), \tag{4}$$

where δ_{MHE} is the estimated material thickness of inflatable elements; δ_{MPE} is the calculated material thickness of deployable elements; δ_0 is the minimum thickness of the film material at which the shell maintains its integrity under the influence of internal pressure, since the internal pressure in the ADS shell is several times of magnitude higher than the external pressure of the Earth's atmosphere, the deforming moments can be neglected, thus the minimum thickness of the film material is calculated using known expressions from the theory of momentless shells; δ_C is the thickness of the material that will be removed from the shell due to the influence of space vacuum (sublimation); δ_A is the thickness of the material that will be removed from the shell due to the influence of atomic oxygen; $\bar{\delta}$ is the calculation error, which is determined using the expression:

$$\bar{\delta} = \delta_{БК} + \delta_{F_{10.7}} + \delta_{AK}, \tag{5}$$

$\delta_{БК}$ is the error to determine the ballistic coefficient, according to [13, 14], we take (600 – 1000) km for the entire range of orbital altitudes; $\delta_{F_{10.7}}$ is the error to forecast the solar activity index, according to [15, 16], we take (600 – 1000) km for the entire range of orbital altitudes; δ_{AK} is the error to determine the concentration of atomic oxygen, according to [17, 18], we take (600 – 750) km for the range of orbital altitudes, (750 – 1000) km for the range of orbital altitudes.

Thus, the coefficient of error to determine the material thickness of inflatable and deployable elements for the range of orbital altitudes of 600-750 km will be equal to $\delta_{AK} = 0,02$, and for the range of orbital altitudes of 750-1000 km will be equal to $\delta_{AK} = 0,25$.

For a spherical film element, the minimum thickness of the film material δ_0 will be defined as [19]:

$$\delta_0 = \frac{P_{HT} r_C}{2\sigma}, \tag{6}$$

where P_{HT} is the excess pressure in the shell of the inflatable element of the ADS; r_C is the radius of the spherical inflatable element; σ is the tensile strength of the material, according to [20], for the polyimide material, we accept $\sigma = 7{,}5 \cdot 10^7$ Па.

For a cylindrical film element, the minimum thickness of the film material δ_0 will be determined as [19]:

$$\delta_0 = \frac{P_{HT} r_{Ц}}{2\sigma}, \tag{7}$$

where $r_{Ц}$ is the radius of the cylindrical inflatable element.

For a toroidal film element, the minimum thickness of the film material δ_0 will be determined as [19]:

$$\delta_0 = \frac{P_{HT} r_{CT} \left(2r_T - r_{CT}\right)}{2\sigma\left(r_T - r_{CT}\right)}, \tag{8}$$

where r_{CT} is the cross-sectional radius of the toroidal inflatable element; r_T is the radius of the toroidal inflatable element.

The impact of the space vacuum on the polymer material leads to its loss, primarily due to its sublimation. The rate of change in the thickness of the polymer material under the influence of sublimation δ_C is determined by the expression [21]:

$$\delta_C = \int_{t_0}^{t_L} S_П \frac{p_{THГ}}{\rho_{ПМ}} \sqrt{\frac{\mu_{ПМ}}{2\pi N_A k_Б T_{ППМ_i}}} dt, \tag{9}$$

where $S_П$ is the surface area of the ADS; $p_{THГ}$ is the saturated gas pressure of the sublimated material is determined by the formula [22]:

$$p_{THГ} = 0{,}0007181 \cdot e^{\left(A - \frac{B}{T_{ППМ_i}}\right)}, \tag{10}$$

A, B are the coefficients, taken according to [23] to be equal to $A = 3, B = 3000$; $\rho_{ПМ}$ is the density of sublimated film material; $\mu_{ПМ}$ is the molecular mass of the film material; N_A is the Avogadro's number, $N_A = 6{,}022 \cdot 10^{23}\,\text{mol}^{-1}$; $k_Б$ is the Boltzmann constant, $k_Б = 1{,}38 \cdot 10^{-23}$ J/K; $T_{ППМ_i}$ – temperature of the film material surface, which is calculated according to the formulas [24] in the shady and sunny parts of the orbit:

$$T_{ППМ_0} = \sqrt[4]{\frac{S_З J_З}{S_П \sigma} + \frac{S_A J_A}{S_П}\left(\frac{\alpha}{\varepsilon}\right)}, \tag{11}$$

$S_З$ is the projection area of the spacecraft onto the plane perpendicular to the direction of the Earth's radiation, for unoriented flight, we take it equal to the area of the middle cross-section S_M; $J_З$ is the intensity of the Earth's radiation; α is the material absorption coefficient; ε is the emissivity of the material; $S_П$ is the surface area of the spacecraft; σ is the Stefan Boltzmann constant, $\sigma = 5,67 \cdot 10^{8}\ \mathrm{Wm^2K^4}$; S_A is the projection area of the spacecraft on the plane perpendicular to the direction of the radiation of the Sun reflected from the Earth; J_A is the intensity of the radiation of the Sun reflected from the Earth;

$$T_{ППМ_1} = \sqrt[4]{\frac{S_З J_З}{S_П \sigma} + \frac{(S_A J_A + S_C J_C)}{S_П}\left(\frac{\alpha}{\varepsilon}\right)}, \quad (12)$$

S_C is the projection area of the spacecraft on the plane perpendicular to the direction of the Sun's radiation; J_C is the intensity of the Sun's radiation.

As is known [24], the environment around the space vehicle during its movement at altitudes of up to ≈800 km is aggressive towards polymer films and coatings of the space vehicle. In near-Earth orbits, the factors that determine the change in the chemical, thermo-optical and mechanical properties of polymers are high-speed atomic oxygen flows. The change in film thickness under the influence of atomic oxygen is determined by the formula:

$$\delta_A = \int_{t_0}^{t_L} R_E \cdot \Phi_{AK} dt, \quad (13)$$

where R_E is the volumetric coefficient of film mass loss, which is determined by the formula [25]:

$$R_E = 10^{-30} \cdot \left(9,5 - 8,3 \cdot e^{0,15 \cdot (1-\gamma)}\right), \quad (14)$$

γ is the erosion coefficient; Φ_{AK} is the atomic oxygen flow is found from the expression [25]:

$$\Phi_{AK} = \rho_{O_i} \cdot V_{KA_i}, \quad (15)$$

ρ_{O_i} is the atomic oxygen concentration at the orbit altitude h_i; V_{KA_i} is the spacecraft velocity at the orbit altitude h_i, for a circular orbit is determined using the expression:

$$V_{KA_i} = \sqrt{\frac{\mu}{r_i}}, \quad (16)$$

r_i is the radius vector of the spacecraft; h_i is the altitude of the spacecraft orbit;

$$h_i = r_i - R_З, \quad (17)$$

$R_З$ is the radius of the Earth, $R_З = 6371\,\mathrm{km}$.

For the ADS class using the transformation of the structural elements of space objects, the AE parameters are not calculated, since it is assumed that the functions of the AE will be performed by the transformed structural elements of the space objects.

Monoblock systems parameters. Structurally, monoblock ADSs consist of AE, IS and SS. In this work, a sphere is chosen as a typical AE of a system of this class. The "Sphere" ADS configuration consists of a spherical AE. So, AE parameters of this configuration are: diameter of AE $d_{СФ}$; volume of AE $V_{СФ}$; mass of AE m_{AE} .

The diameter of the sphere can be expressed in terms of the area of the median cross-section S_M , which for unoriented flight is equal to:

$$S_M = \frac{S_П}{4}, \quad (18)$$

$$S_П = 3{,}141593 \cdot d_{СФ}^2, \quad (19)$$

$$d_{СФ} = 1{,}13\sqrt{S_M}, \quad (20)$$

where $S_П$ is the surface area of the sphere.

The volume of the sphere is determined from the ratio:

$$V_{СФ} \approx 0{,}5236 d_{СФ}^2 = 0{,}755\sqrt{S_M^3}. \quad (21)$$

The mass of AE is determined using the formulas

$$m_{AE} = 4 S_M \delta_{MHE} \rho_{MHE}, \quad (22)$$

where ρ_{MHE} is the material density of inflatable elements; δ_{MHE} is the material thickness of an inflatable element.

Frame-inflatable systems parameters. Frame-inflatable ADSs structurally consist of a combination of inflatable elements (for example, inflatable masts) and a membrane made of thin-film elastic material.

The following configurations of frame-inflatable ADSs are considered in the work:

- "Round Shield";
- "Two dihedral panels";
- "Prism Triangle";
- "Prism Square";
- "A cone made of tori";
- "A cone made of tori with spheres placed inside";
- "Bulk sail".

The configuration of the "Round shield" ADS consists of 4 inflatable elements (3 inflatable masts and a torus shell) and one deployable element (a flat round shield).

The AE parameters of this configuration are: shield diameter of AE $d_Щ$; volume of inflatable elements V_{HE} mass of AE m_{AE} .

The initial data to calculate the mass of the ADS in the form of a flat round shield are: the ADS median cross-section area S_M ; length of the inflatable masts l_{HM} , which is equal to the diameter of the round shield $d_Щ$; diameter of inflatable masts d_{HM} depends on the length of the inflatable mast l_{HM} and is defined as $d_{HM} = 0{,}035 l_{HM}$.

The mass of AE m_{AE} in the form of a flat round shield is determined by the ratios:

$$m_{AE} = m_{HE} + m_{PE}, \quad (23)$$

$$m_{HE} = 1,224 S_M \delta_{MHE} \rho_{MHE}, \quad (24)$$

$$m_{PE} = 2,776 S_M \delta_{MPE} \rho_{MPE}. \quad (25)$$

The volume of inflatable elements is calculated as follows:

$$V_{HE} = V_T + 3V_{HC} \approx 0,01\sqrt{S_M^3}, \quad (26)$$

where V_T is the volume of the torus shell; V_{HC} is the volume of the inflatable sling.

The "Two dihedral panels" ADS configuration consists of 2 inflatable elements (2 inflatable mast-ribs) and 2 deployable elements (4 faces).

The AE parameters of this configuration are: length of the inflatable mast l_{HM}, by which the length of panel $l_{ДП}$ is determined; diameter of the inflatable mast d_{HM}, it has been accepted $d_{HM} = 0,035 l_{ДП}$; width $a_{ДП}$ of the polymer material web of each dihedral panel, while $l_{HM} = l_{ДП}$ and $a_{ДП} = 0,12 l_{ДП}$; mass of AE m_{AE}.

The mass of AE m_{AE} in the form of two dihedral panels is determined by the ratios:

$$m_{AE} = m_{HE} + m_{PE}, \quad (27)$$

$$m_{HE} = 0,531 S_M \delta_{MHE} \rho_{MHE}, \quad (28)$$

$$m_{PE} = 3,469 S_M \delta_{MPE} \rho_{MPE}. \quad (29)$$

The volume of inflatable elements V_{HE} is calculated using the formulas:

$$V_{HE} = 2V_{HM} \approx 0,011\sqrt{S_M^3}, \quad (30)$$

where V_{HM} is the volume of inflatable mast.

The ADS configuration "Prism Triangle" consists of 6 inflatable elements (3 inflatable masts-ribs of the pyramid and 3 inflatable masts-sides of the base of the pyramid) and 3 deployable elements (3 faces of the pyramid).

The initial data to calculate the mass of the AE in the form of a three-sided pyramid are: diameter of inflatable masts d_{HM} depends on the length of the inflatable mast l_{HM} and is defined as $d_{HM} = 0,035 l_{HM}$; flat angle A at the top of the pyramid is 60°; area of the median cross-section S_M of the ADS. The AE parameters of this configuration are: length of the base of the pyramid a_O; mass of AE m_{AE}.

The mass of AE in the form of a three-sided pyramid is determined by the relations:

$$m_{AE} = m_{HE} + m_{PE}, \quad (31)$$

$$m_{HE} = S_{ПHE}\delta_{ПHE}\rho_{ПHE}\,, \quad (32)$$

$$m_{PE} = S_{ПPE}\delta_{ПPE}\rho_{ПPE}\,, \quad (33)$$

$$S_{ПHE} = S_{HM} = 6\left(2\cdot\pi\cdot 0,0175 a_O^2\right) = 0,66 a_O^2\,, \quad (34)$$

where a_O is the length of the base side of the triangular pyramid; S_{HM} is the surface area of inflatable masts;

$$S_{ПPE} = S_{Г} = \frac{p_2 A_2}{2} = 1,305 a_O^2\,, \quad (35)$$

$S_{Г}$ is the area of the faces of the pyramid; p_2 is the perimeter of the base of the pyramid

$$p_2 = 3a_O\,, \quad (36)$$

A_2 is the length of the apophemum (height of the face) of the pyramid

$$A_2 = \sqrt{a_O^2 - \frac{a_O^2}{4}} = 0,87 a_O\,. \quad (37)$$

Let's express the length of the base of the pyramid in terms of the area of the median cross-section of the ADS:

$$S_M = \frac{S_П}{4}\,, \quad (38)$$

$$S_П = S_{ПHE} + S_{ПPE} = 1,965 a_O^2\,, \quad (39)$$

$$a_O = 1,427\sqrt{S_M}\,. \quad (40)$$

The volume of inflatable elements V_{HE} is calculated using the formulas:

$$V_{HE} = 6V_{HM} \approx 0,017\sqrt{S_M^3}\,, \quad (41)$$

where V_{HM} – the volume of inflatable mast.

The configuration of the ADS "Prism square" consists of 8 inflatable elements (4 inflatable masts-ribs of the pyramid and 4 inflatable masts-sides of the base of the pyramid) and 4 deployable elements (4 faces of the pyramid).

The initial data to calculate the mass of the ARS in the form of a four-sided pyramid are: diameter of inflatable masts d_{HM} depends on the length of the inflatable mast and is defined as $d_{HM} = 0,035 l_{HM}$; flat angle A at the top of the pyramid is 60°; area of the median cross-section of the ARS S_M. The AE parameters of this configuration are: length of the base of the pyramid a_O; mass of AE.

The mass of AE in the form of a triangular prism is determined by the relations:

$$m_{AE} = m_{HE} + m_{PE}\,, \quad (42)$$

$$m_{HE} = S_{ПHE}\delta_{ПHE}\rho_{ПHE}\,, \quad (43)$$

$$m_{PE} = S_{ПРЕ} \delta_{ПРЕ} \rho_{ПРЕ} , \tag{44}$$

$$S_{ПНЕ} = S_{HM} = 8\left(2 \cdot \pi \cdot 0{,}0175 a_O^2\right) = 0{,}88 a_O^2 , \tag{45}$$

where a_O is the length of the side of the base of the triangular pyramid; S_{HM} is the area of inflatable masts;

$$S_{ПРЕ} = S_{Г} = \frac{p_2 A_2}{2} = 1{,}74 a_O^2 , \tag{46}$$

$S_{Г}$ is the area of the faces of the pyramid; p_2 is the perimeter of the base of the pyramid

$$p_2 = 4 a_O , \tag{47}$$

A_2 is the length of the apophemum (height of the face) of the pyramid

$$A_2 = \sqrt{a_O^2 - \frac{a_O^2}{4}} = 0{,}87 a_O . \tag{48}$$

Let's express the length of the base of the pyramid in terms of the area of the median cross-section of the ARS:

$$S_M = \frac{S_{П}}{4} , \tag{49}$$

$$S_{П} = S_{ПНЕ} + S_{ПРЕ} = 1{,}965 a_O^2 , \tag{50}$$

$$a_O = 1{,}235 \sqrt{S_M} . \tag{51}$$

The volume of inflatable elements V_{HE} is calculated using the formulas:

$$V_{HE} = 6 V_{HM} \approx 0{,}015 \sqrt{S_M^3} , \tag{52}$$

where V_{HE} is the volume of inflatable mast.

ADS "Cone made of tori" is made in the form of a conical shell made of tori shells. The number of tori is selected based on the conditions to ensure the required area of the median cross-section. The ADS parameters of this configuration will be: diameter of the 1st torus d_{T_1} ; diameter of the nth torus d_{T_n} ; mass of AE m_{AE} . The mass of AE in the form of a conical shell composed of toric shells is determined by the following relations:

$$m_{AE} = m_{HE} , \tag{53}$$

$$m_{HE} = S_{ПНЕ} \delta_{ПНЕ} \rho_{ПНЕ} , \tag{54}$$

$$S_{ПНЕ} = S_{ПТО} + S_{ПНС} , \tag{55}$$

where $S_{ПТО}$ is the surface area of the torus shells

$$S_{ПТО} = \sum_{n=1}^{i} S_{ПТО_n} , \tag{56}$$

n is the torus shell number; i is the number of torus shells in AE;

$$S_{ПТО_1} = 19,74 d_{КО}, \tag{57}$$

$d_{КО}$ is the diameter of the removed spacecraft;

$$S_{ПТО_n} = 9,87 d_{T_n} d_{КО}, \tag{58}$$

d_{T_n} is the diameter of the nth torus shell;

$$d_{T_n} = d_{T_{n-1}} + 0,74 d_{КО}, \tag{59}$$

$S_{ПНС}$ is the surface area of inflatable slings

$$S_{ПНС} = 3 \cdot 3,141593 d_{НС} l_{НС}, \tag{60}$$

$d_{НС}$ is the diameter of the inflatable slings, we accept $d_{НС} = 0,1 d_{КО}$; $l_{НС}$ is the length of the removed spacecraft, we accept $l_{НС} = 0,5 d_{КО}$.

ADS "Cone made of tori, with spherical shells placed inside" are made in the form of a conical shell, made of toric shells, inside which spherical shells are placed. The number of tori is also selected based on the conditions for ensuring the required cross-section area. The ADS parameters of this configuration will be: diameter of the 1st torus d_{T_1}; diameter of the nth torus d_{T_n}; the diameter of the spherical shell placed inside the torus $d_{СФ}$; the number of torus shells n; number of spherical shells m; mass of AE m_{AE}. The mass of an aerodynamic element in the form of a cone made of tori, with spherical shells placed inside is determined by the following ratios:

$$m_{AE} = m_{HE}, \tag{61}$$

$$m_{HE} = S_{ПНЕ} \delta_{ПНЕ} \rho_{ПНЕ}, \tag{62}$$

$$S_{ПНЕ} = S_{ПТО} + S_{ПНС} + S_{ПС}, \tag{63}$$

where $S_{ПТО}$ is the surface area of the torus shells, which is determined using ratios (56 – 59); $S_{ПНС}$ is the surface area of inflatable slings is defined using an expression (60); $S_{ПС}$ is the surface area of the spherical shells, which are placed inside the toric shells, is determined as follows:

$$S_{ПНС} = \sum_{n=1}^{i} \left(m_n \pi d_{СФ}^2 \right), \tag{64}$$

where n is amount of torus shells; i is the number of torus shells; $d_{СФ}$ is the diameter of the spherical shell is defined as follows:

$$d_{СФ} = \sqrt{\frac{4S_M - \pi d_{КО}}{12\pi}}, \tag{65}$$

S_M is the ADS cross-sectional area; $d_{КО}$ is the diameter of deorbiting space object, for a cylindrical shape, we will apply that in the diameter of its base, for the shape of a cube and a parallepiped, it will be added that in the diameter of the circle described above; m_n is the number of spherical shells of the nth torus shell is calculated using the scheme shown in Fig. 2 and the following ratios:

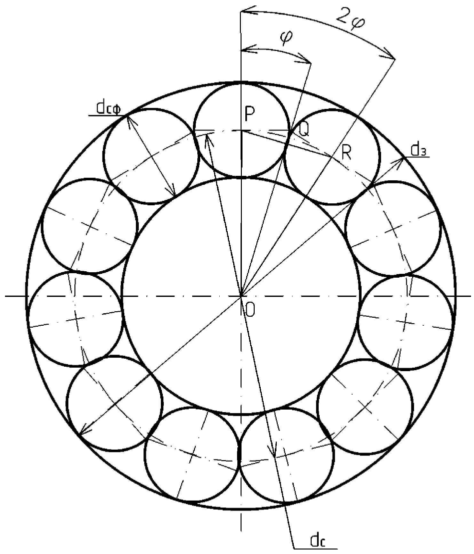
$$m_n = \frac{\pi\left(d_{n_{зов}} + d_{n_{вн}}\right)}{d_{n_{зов}} - d_{n_{вн}}}, \quad (66)$$

$d_{n_{зов}}$ is the outer diameter of the nth torus shell is determined as follows:

$$d_{n_{зов}} = d_n + d_{СФ}, \quad (67)$$

d_n is the diameter of the nth torus; $d_{n_{вн}}$ is the inner diameter of the nth torus shell is calculated using the expression:

$$d_{n_{вн}} = d_n - d_{СФ}. \quad (68)$$



d_C – cross-sectional diameter of the torus; $d_{СФ}$ – diameter of the spherical shell, which is placed in the torus shell; d_3 – diameter of the torus; φ – angle of placement of spherical shells in the torus shell

Fig. 2 – Structural diagram for calculating the number of spherical shells

ADS "Bulk sail" structurally consists of 3 round membranes, which are placed orthogonally to each other, to which are attached toric shells, inside which spherical shells are placed. To increase the rigidity of the structure, inflatable masts are orthogonally attached to the diameter of the shield, which also contain spherical shells inside. Let us assume that the cross-sectional diameter of the torus shell d_{TO} coincides with the cross-sectional diameter of the inflatable mast $d_{НЩ}$ and is defined as $d_{TO} = d_{НЩ} = 0{,}035 d_M$, where d_M is the diameter of the membrane of the round shield. The AE parameters of this ADS configuration are the following list: the diameter of the membrane of the round shield d_M; the mass of the aerodynamic element m_{AE}.

The mass of the aerodynamic element m_{AE} is determined using the expressions:

$$m_{AE} = m_{HE} + m_{PE}, \quad (69)$$

where $m_{НЕ}$ is the mass of inflatable elements; $m_{РЕ}$ is the mass of deployable elements.

The mass of inflatable elements is defined as:

$$m_{НЕ} = m_{НЩ} + m_{ТО}, \tag{70}$$

$$m_{НЩ} = 3S_{ПНЩ} \cdot ¢_{МНЩ} \cdot °_{МНЩ} + n_{СНЩ} S_{ПСНЩ} \cdot ¢_{МСНЩ} \cdot °_{МСНЩ}, \tag{71}$$

$$S_{ПНЩ} = 0,11 d_{М}^{2}, \tag{72}$$

$$n_{СНЩ} = 3\frac{d_{М}}{0,035 d_{М}} \approx 87, \tag{73}$$

$$S_{ПСНЩ} = 0,00385 d_{М}^{2}, \tag{74}$$

$$m_{ТО} = 3S_{ПТО} \cdot ¢_{МТО} \cdot °_{МТО} + n_{СФТО} \cdot S_{ПСТО} \cdot ¢_{МСТО} \cdot °_{МСТО}, \tag{75}$$

$$S_{ПТО} = 0,3575 d_{М}^{2}, \tag{76}$$

$$n_{СТО} = 3\frac{®\left(d_{М} + 0,035 d_{М}\right)}{0,035 d_{М}} \approx 279, \tag{77}$$

$$S_{ПСТО} = 0,00385 l_{НЩ}^{2}, \tag{78}$$

where $m_{НЕ}$ is the mass of inflatable elements; $m_{НЩ}$ is the mass of inflatable masts; $m_{ТО}$ is the mass of toric shells; $S_{ПНЩ}$ is the surface area of the inflatable mast; $¢_{МНЩ}$ is the thickness of the material of the inflatable mast; $°_{МНЩ}$ is the density of the material of the inflatable mast; $n_{СНЩ}$ is the number of spherical shells placed in inflatable masts; $S_{ПСНЩ}$ is the surface area of the spherical shell, which is placed in the inflatable mast; $¢_{МСНЩ}$ is the thickness of the material of the spherical shell, which is placed in the inflatable mast; $°_{МСНЩ}$ is the density of the material of the spherical shell, which is placed in the inflatable mast; $S_{ПТО}$ is the surface area of the torus shell; $¢_{МТО}$ is the thickness of the material of the torus shell; $°_{МТО}$ is the density of the material of the torus shell; $n_{СТО}$ is the number of spherical shells that are placed in toric shells; $S_{ПСТО}$ is the surface area of the spherical shell, which is placed in the torus shell; $¢_{МСТО}$ is the thickness of the material of the spherical shell, which is placed in the torus shell; $°_{МСТО}$ is the density of the material of the spherical shell, which is placed in the torus shell.

Also, in its composition, the aerodynamic element has the following 3 round membranes. The mass of the deployable elements $m_{РЕ}$ is determined as follows:

$$m_{РЕ} = 3S_{ПМ} \cdot ¢_{ММ} \cdot °_{ММ}, \tag{79}$$

$$S_{ПМ} = 0,7854 d_{М}^{2}, \tag{80}$$

where $S_{ПМ}$ is the membrane surface area; $¢_{ММ}$ is the thickness of the membrane material; $°_{ММ}$ is the density of the membrane material.

It is assumed that the deorbiting space object with the help of this system, will move around the orbit in a non-orientational way. For non-orientational orbital motion, it was expressed the medial cross-section area S_M of the aerodynamic element of the ADS using the following expression:

$$S_M = \frac{S_{ПАЕ}}{4} = 1{,}5287 d_M^2, \tag{81}$$

where $S_{ПАЕ}$ is the total surface area of the aerodynamic element.

Next, we will express the diameter of the membrane in terms of the area of the average cross-section of the aerodynamic element of the ADS:

$$d_M = 0{,}81\sqrt{S_M}\ . \tag{82}$$

Parameters of deployable systems. Deployable ADSs are made in the form of square sails. The sail consists of 4 retractable masts and a fabric canvas. The AE parameters of ADS this configuration: length of the base of the sail $a_П$; mass of AE m_{AE} .

The mass of AE is determined using ratios

$$m_{AE} = 4S_M ¢_{MPE}\, °_{MPE} + \overline{m_{BE}} \cdot 2{,}83\sqrt{S_M}, \tag{83}$$

where S_M is the medial cross-section area of the AE; $°_{MPE}$ is the material density of deployable elements; $¢_{MPE}$ is the material thickness of deployable elements; $\overline{m_{BE}}$ is the linear density (mass per unit length) of retractable elements, we take $\overline{m_{BE}} = 0{,}025\,кг/м$.

1.4. Parameters of the aerodynamic element of the transformed aerodynamic deorbit systems. When using the method of transforming the structure of a space object into an aerodynamic system, the mass of the system will be determined by the mass of the deployment system

$$m_{ACB} = m_{CP}\,. \tag{84}$$

In order to calculate the limits of applicability of this method, the area of the median cross-section in this case will be determined by the overall characteristics of the spacecraft:

$$S_M = 0{,}25\left(S_{ВПКО} + S_{ПКО}\right), \tag{85}$$

where $S_{ВПКО}$ is the area of the side panels of the spacecraft; $S_{ПКО}$ is the total surface area of the spacecraft.

For the option of the spacecraft layout in the form of a square prism, the area of the side panels will be determined as:

$$S_{ВПКО} = 4a^2, \tag{86}$$

where a is the length of the face of the side panel.

The total surface area of the spacecraft in this case will be determined as:

$$S_{ПКО} = 6a^2. \tag{87}$$

For the option of the spacecraft layout in the form of a rectangular prism, the area of the side panels will be determined:

$$S_{БПКО} = 4ab, \tag{88}$$

where b is the width of the face of the side panel.

The total surface area of the spacecraft in this case will be determined as:

$$S_{ПКО} = 2a^2 + 4ab. \tag{89}$$

Conclusions. The aerodynamic systems were decomposed. As a result of the decomposition, the hierarchical structure of the system is determined, which has the following levels: subsystem level, element level, parameter level. The analysis of materials for the manufacture of structural elements of the aerodynamic element of the system is carried out. A set of design parameters for aerodynamic systems has been formed. This set was used in the development of a mathematical model for selecting the parameters of the aerodynamic element of deorbiting systems of various classes, namely: monoblock, frame-inflatable, systems formed by transforming the structure of a space object into an aerodynamic system, deployable systems. The model for determining the material thickness takes into account the following factors: the effect of space vacuum and atomic oxygen on the shell material, the effect of overpressure on the shell. The model also takes into account errors in determining the ballistic coefficient of the aerodynamic system with the space object being deorbited, in determining the solar activity index, and in determining the atomic oxygen concentration. This mathematical model for selecting aerodynamic system parameters allows to build nomograms for determining the parameters of space objects' deorbiting systems of different mass classes and dislocation orbits.

Published on the basis of the agreement №363-9 from 09.01.2023 on scientific and technical cooperation between ITM NASU and SSAU and Northwestern Polytechnical University of Xi'an, People's Republic of China.

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Received on August 21, 2023,
in final form on September 28, 2023