, , ; e-mail: avramov@nas.gov.ua

The model of geometrically nonlinear behavior of functionally graded composite cylindrical shell with nanotubes reinforcements is derived. The simply supported cylindrical shell is treated. The Reddy high-order shear deformation theory is used. Three projections of the displacements and two rotations angles of middle surface normal are the main unknowns of this problem. The potential energy of the cylindrical shell geometrically nonlinear deformation is derived with account of shear. Three displacements projections and two rotation angles of the middle surface normal is expanded using the eigenmodes of the cylindrical shell vibrations. The axisymmetric eigenmodes are accounted in these expansions too. High dimension nonlinear system of ordinary differential equation is obtained to describe the structure nonlinear vibrations using the assumed-mode method. The piston theory is used to describe the supersonic gas theory. The extended rule of mixture is used to obtain the mechanical features of nano composites. The calculation of the characteristic exponents and the direct numerical integrations of the motions equations are used to analyze the dynamic stability of the trivial equilibrium. As a result of the numerical analysis, it is obtained, that the trivial equilibrium loses stability due to the Hopf bifurcation. The limit

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cycle is originated due to the Hopf bifurcation. This cycle describes the travelling waves of the cylindrical shell. The harmonic balanced method is used to analyze the limit cycle behavior, when the flow pressure is varied. The mono harmonic approximation of the self-sustained vibrations is used. The data, which are obtained by the harmonic balanced method, are compared with the results of the direct numerical integrations. The results, which are obtained by two methods, are close. Therefore, the harmonic balanced method are true for self- sustained vibrations analysis.









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(*x*, *θ*, *z***)** (. 1).

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UD-C	CNT	$V_{CNT}(z) = V_{CNT}^*;$
FGV-	CNT	$V_{CNT}(z) = \left(1 + \frac{2z}{h}\right) V_{CNT}^{\bullet};$
FG A -0	CNT	$V_{CNT}(z) = \left(1 - \frac{2z}{h}\right) V_{CNT}^{\star};$
FGX-	CNT	$V_{CNT}(z) = \frac{4 z }{h} V_{CNT}^*;$
FGO-	CNT	$V_{CNT}(z) = 2\left(1 - \frac{2 z }{h}\right) V_{CNT}^*.$

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 $E_{11}(z) = \eta_{1}V_{CNT}(z)E_{11}^{CNT} + V_{m}(z)E^{m}; E_{22}(z) = \frac{\eta_{2}E_{22}^{CNT}E^{m}}{V_{CNT}(z)E^{m} + V_{m}(z)E_{22}^{CNT}}G_{12}(z) = \frac{\eta_{3}G_{12}^{CNT}G^{m}}{V_{CNT}(z)G^{m} + V_{m}(z)G_{12}^{CNT}};$ $E_{11}^{CNT}, E_{22}^{CNT}, G_{12}^{CNT} - ; \mu_{12}^{CNT}, \eta_{3} - ; E^{m}, G^{m} - ; P^{CNT}, \rho^{m} - ; P^{CNT}, \rho^{m} - ; P^{CNT}, \rho^{m} - ; \rho^{CNT}, \rho^{m} - ; \rho^{2NT}, \rho^{m} - ; P_{12}^{CNT}, \rho^{m} - ; P_{12}^{CN}, \rho$

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Ζ

$$\begin{split} \gamma_{XZ}, \gamma_{GZ} &- \qquad ; \quad \mathcal{E}_{XX}, \mathcal{E}_{GG}, \gamma_{ZZ}, \gamma_{XZ}, \gamma_{XG} - \\ ; \quad \sigma_{XZ}, \sigma_{GZ} - \qquad ; \quad \sigma_{XX}, \sigma_{GG}, \sigma_{XG} - \\ \\ x, \theta &= z \\ u_x(x, \theta, t, z) = u(x, \theta, t) + z\phi_1(x, \theta, t) + z^2\psi_1(x, \theta, t) + z^2\gamma_1(x, \theta, t); \quad (3) \\ u_g(x, \theta, t, z) = u(x, \theta, t) + z\phi_1(x, \theta, t) + z^2\psi_1(x, \theta, t) + z^2\gamma_2(x, \theta, t); \quad (3) \\ u_g(x, \theta, t, z) = (1 + \frac{z}{R})v(x, \theta, t) + z\phi_2(x, \theta, t) + z^2\psi_2(x, \theta, t) + z^2\gamma_2(x, \theta, t); \\ R - & u_x(x, \theta, t, z) = w(x, \theta, t), \\ \psi_1(x, \theta, t), v(x, \theta, t), \psi_1(x, \theta, t), \psi_2(x, \theta, t), \phi_1(x, \theta, t) + z^2\gamma_2(x, \theta, t) \\ \psi_1(x, \theta, t), \gamma_1(x, \theta, t), \psi_2(x, \theta, t), \phi_2(x, \theta, t) \\ (4), & (3) \\ \\ T = 0.5 \int_0^{2\pi} \int_0^L \int_{0.5h}^{0.5h} \rho(z)(u_x^2 + u_g^2 + u_g^2)(1 + \frac{x}{R})dzdxRd\theta, \quad (6) \\ L - & (3) \\ (6), \\ T = 0.5 \int_0^{2\pi} \int_u^L \{r_9(u^2 + v^2 + w^2) + 2r_1(\frac{v^2}{R} + u\phi_1 + v\phi_2) + \\ + r_2(\frac{v^2}{R^2} + \phi_1^2 + \phi_2^2 + \frac{x}{R}v\phi_1 + 2v\psi_2) + 2r_3(u_Y^2 + \frac{v}{R} + \varphi_2 + \phi_2 \psi_2) + \\ r_1 = \int_{0.5h}^{0.5h} z^i \rho(z)dz; \quad i = 0, \dots, 6. \\ (7) \\ 2 \prod v = \int_0^L \int_0^{2\pi} \int_{-0.5h}^{0.5h} (\sigma_{Xx} \varepsilon_{Xx} + \sigma_{Gg} \varepsilon_{Gg} + \sigma_{gZ} \gamma_{GZ} + \sigma_{XZ} \gamma_{XZ} + \sigma_{XG} \gamma_{Xg})(1 + \frac{z}{R})dz. \end{split}$$

(2)

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$$\prod \square = \frac{1}{2} \int_0^L \int_0^{2\pi} \int_{-0.5h}^{0.5h} \left(Q_{11} \varepsilon_{XX}^2 + 2Q_{12} \varepsilon_{\theta\theta} \varepsilon_{XX} + Q_{22} \varepsilon_{\theta\theta}^2 + G_{23} \gamma_{\theta Z}^2 + G_{13} \gamma_{XZ}^2 + G_{12} \gamma_{X\theta}^2 \right) \left(1 + \frac{z}{R} \right) dz R d\theta dx.$$

$$\varepsilon_{XX}, \varepsilon_{\theta\theta}, \gamma_{X\theta}$$

$$u_X, u_{\theta}, u_Z :$$

$$\varepsilon_{XX} = \frac{\partial u_X}{\partial x} + 0.5 \left[\left(\frac{\partial u_X}{\partial x} \right)^2 + \left(\frac{\partial u_{\theta}}{\partial x} \right)^2 + \left(\frac{\partial u_Z}{\partial x} \right)^2 \right];$$

$$\varepsilon_{\theta\theta} = \frac{1}{R(1 + zR^{-1})} \left(\frac{\partial u_{\theta}}{\partial \theta} + u_Z \right) +$$

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$$\gamma_{X\theta} = \frac{1}{R(1+zR^{-1})} \frac{\partial u_X}{\partial \theta} + \frac{\partial u_\theta}{\partial x} + \frac{1}{R(1+zR^{-1})} \left\{ \frac{\partial u_X}{\partial \theta} \frac{\partial u_X}{\partial x} + \left(\frac{\partial u_\theta}{\partial \theta} + u_Z \right) \frac{\partial u_\theta}{\partial x} - \frac{\partial u_Z}{\partial x} \left(-\frac{\partial u_Z}{\partial \theta} + u_\theta \right) \right\}.$$
(10)

$$\begin{split} \varepsilon_{XX} &= \varepsilon_{XX,0} + zk_{XX}^{(0)} + z^2 k_{XX}^{(1)} + z^3 k_{XX}^{(2)};\\ \varepsilon_{\theta\theta} &= \varepsilon_{\theta\theta,0} + zk_{\theta\theta}^{(0)} + z^2 k_{\theta\theta}^{(1)} + + z^3 k_{\theta\theta}^{(2)};\\ \gamma_{X\theta} &= \gamma_{X\theta,0} + zk_{X\theta}^{(0)} + z^2 k_{X\theta}^{(1)} + z^3 k_{X\theta}^{(2)};\\ \gamma_{\theta Z} &= \gamma_{\theta Z,0} + zk_{\theta Z}^{(0)} + z^2 k_{\theta Z}^{(1)} + z^3 k_{\theta Z}^{(2)}, \end{split}$$
(11)

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$$\prod \Box = \mathbf{0.5} \int_0^L \int_0^{2\pi} \left(\sum_{\nu=0}^6 \prod \widetilde{\Box}_{\nu} \right) dx R d\theta,$$
(12)

$$\begin{split} \Pi \widetilde{\mathbb{i}}_{0} &= Q_{11}^{(0)} \varepsilon_{XX,0}^{2} + 2Q_{12}^{(0)} \varepsilon_{\theta\theta,0} \varepsilon_{XX,0} + Q_{22}^{(0)} \varepsilon_{\theta\theta,0}^{2} + G_{23}^{(0)} \gamma_{\thetaZ,0}^{2} + G_{12}^{(0)} \gamma_{X\theta,0}^{2} + G_{13}^{(0)} \gamma_{XZ,0}^{2}; \\ \Pi \widetilde{\mathbb{i}}_{1} &= 2Q_{11}^{(1)} \varepsilon_{XX,0} k_{XX}^{(0)} + 2Q_{22}^{(1)} \varepsilon_{\theta\theta,0} k_{\theta\theta}^{(0)} + 2Q_{12}^{(1)} \varepsilon_{\theta\theta,0} k_{XX}^{(0)} + \end{split}$$

 $\Pi \widetilde{\mathbb{I}}_{2} = 2Q_{12}^{(2)} k_{\theta\theta}^{(0)} k_{XX}^{(0)} + 2G_{12}^{(2)} \gamma_{X\theta,0} k_{X\theta}^{(1)} + Q_{22}^{(2)} k_{\theta\theta}^{(0)2} + 2G_{13}^{(2)} \gamma_{XZ,0} k_{XZ}^{(1)} +$ $\Pi \tilde{\textbf{i}}_{3} = 2G_{12}^{(3)} \gamma_{X\theta,\mathbf{0}} k_{X\theta}^{(2)} + 2G_{12}^{(3)} k_{X\theta}^{(0)} k_{X\theta}^{(1)} + 2G_{23}^{(3)} \gamma_{\theta Z,\mathbf{0}} k_{\theta Z}^{(2)} + 2Q_{12}^{(3)} k_{\theta \theta}^{(2)} \varepsilon_{XX,\mathbf{0}} + 2Q_{12}^{(3)$
$$\begin{split} &\Pi\widetilde{\mathbf{i}}_{4} = Q_{22}^{(4)}k_{\theta\theta}^{(1)2} + G_{23}^{(4)}k_{\theta\theta}^{(1)2} + 2Q_{12}^{(4)}k_{\theta\theta}^{(2)}k_{XX}^{(0)} + 2G_{12}^{(4)}k_{X\theta}^{(0)}k_{X\theta}^{(2)} + 2Q_{22}^{(4)}k_{\theta\theta}^{(0)}k_{\theta\theta}^{(2)} + \\ &+ G_{13}^{(4)}k_{XZ}^{(1)2} + G_{12}^{(4)}k_{X\theta}^{(1)2} + 2Q_{12}^{(4)}k_{\theta\theta}^{(0)}k_{XX}^{(2)} + 2Q_{11}^{(4)}k_{XX}^{(0)}k_{XX}^{(2)}. \end{split}$$

(12),

$$\left(Q_{11}^{(i)}, Q_{12}^{(i)}, Q_{23}^{(i)}, G_{23}^{(i)}, G_{13}^{(i)}, G_{12}^{(i)}\right) = \int_{-0.5h}^{0.5h} z^{i} (Q_{11}, Q_{12}, Q_{22}, G_{23}, G_{13}, G_{12}) dz.$$
(13)

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$$[17]:$$

$$p = -\frac{\gamma p_{m} M^{2}}{\sqrt{M^{2} - 1}} \left[\frac{\partial w}{\partial x} + \frac{M^{2} - 2}{M a_{m} (M^{2} - 1)} \frac{\partial w}{\partial t} - \frac{w}{2R \sqrt{M^{2} - 1}} \right], \quad (14)$$

$$\gamma - \qquad ; p_{m} - \qquad ; a_{m} - \qquad ; a_{m}$$

$$w(x,\theta,t)\sum_{m=1}^{N_1} [A_m \cos(n\theta) + B_m(t)\sin(n\theta)]\sin(\lambda_m x) +$$

+
$$\sum_{m=1}^{N_2} C_{-}(t)\sin(\lambda_{-} \dots x);$$

$$u = \sum_{m=1}^{N_1} [A_{N_1+m}\cos(n\theta) + B_{N_1+m}\sin(n\theta)]\cos(\lambda_m x) + \sum_{m=1}^{N_2} C_{N_2+m}\cos(\lambda_{2m-1}x);$$

$$v(x,\theta,t) = \sum_{m=1}^{N_1} [A_{2N_1+m}(t)\cos(n\theta) + B_{2N_1+m}(t)\sin(n\theta)]\sin(\lambda_m x);$$

$$\phi_{1} = \sum_{m=1}^{N_{1}} \begin{bmatrix} A_{2N_{1}+m}\cos(n\theta) \\ +B_{2N_{1}+m}\sin(n\theta) \end{bmatrix} \cos(\lambda_{m}x) \sum_{m=1}^{N_{2}} C_{2N_{2}+m}\cos(\lambda_{2m-1}x),$$
(15)

$$\mathbf{q} = [A_1, \dots, B_1, \dots, C_1, \dots, A_{3N_1+1}, \dots, B_{3N_1+1}, \dots, C_{2N_2+1}, \dots, C_{3N_2}] = [q_1, \dots, q_{N_*}] - N_* = 10N_1 + 3N_2.$$

. (7)

$$T = T(q_1, ..., q_{N_*}).$$
(15)

$$\prod \square = \prod (q_1, \dots, q_{N_{\bullet}}).$$

$$\sum_{j=1}^{N_{\star}} (m_{ij}\bar{q}_{j} + K_{ij}q_{j}) = \sum_{\nu=1}^{N_{\star}} \sum_{j=1}^{j \le \nu} \alpha_{\nu j}^{(i)} q_{\nu}q_{j} + \sum_{\nu=1}^{N_{\star}} \sum_{j=1}^{j \le \nu} \sum_{j_{1} \le 1}^{j_{1} \le j} \beta_{\nu j j_{1}}^{(i)} q_{\nu}q_{j}q_{j_{1}} + Q_{i}; \quad (16)$$

$$\mathbf{M} = \{m_{ij}\}_{i=1,\dots,N_{\star}}^{j=1,\dots,N_{\star}}; \quad \mathbf{K} = \{K_{ij}\}_{i=1,\dots,N_{\star}}^{j=1,\dots,N_{\star}} - \sum_{j_{1} < 0,\dots,N_{\star}}^{j} \beta_{\nu j j_{1}}^{(i)} - \sum_{j_{1} < 0,\dots,N_{\star}}^{j} \beta_{\nu j_{1}}^{(i)} - \sum_{j_{1} < 0,\dots,N_{\star}}^{j} \beta$$

$$Q_{\nu} = \sum_{m=1}^{N_{1}} (\lambda_{\nu m} q_{m} + \eta_{\nu m} \dot{q}_{m}); \ Q_{3N_{1}+\nu} \sum_{m=1}^{N_{1}} (\lambda_{\nu m} q_{3N_{1}+m} + \eta_{\nu m} \dot{q}_{3N_{1}+m}).$$
(17)

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(16). [14].

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[14].

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(15)

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$$R = 0.25 \text{ M}; h = 5 \cdot 10^{\circ} \text{ M}; \frac{R}{R} = 0.02; L = 1 \text{ M}, \qquad (13)$$
FGV
$$(2) \quad V_{CNT}^{*} = 0.12 .$$

$$M = 5; M = 3 \quad M = 1.5 .$$

$$(15) \quad N_{1} = 2; N_{2} = 1 .$$

$$(16) \quad N_{*} = 23 .$$

M = 3 M = 5 M = 3 M = 5 n = 6

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(18),

 $V_{CNT}^{*} = 0,12$.





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4×4	- 20

	V _{CNT}	n	р _≕ (СR), МПа
	0,12	6	0,33
FGV- NT	0,17	5	0,68
	0,28	5	0,97
	0,12	5	0,38
FGX-CNT	0,17	5	0,76
	0,28	5	1,21
	0,12	6	0,27
FGO-CNT	0,17	5	0,56
	0,28	5	0,71







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n = 27

$\phi_2(x, \theta, t)$

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