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ESTIMATION OF PROBE MEASUREMENTS RELIABILITY IN A SUPERSONIC FLOW OF FOUR-COMPONENT COLLISIONLESS PLASMA

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Метою роботи є оцінка достовірності відновлення концентрації і температури електронів, іонного складу плазми за вольтамперною характеристикою (ВАХ) ізольованої зондової системи з циліндричними електродами. Виконано аналіз побудованої раніше математичної моделі збирання струму зондовою системою при позитивних потенціалах зміщення і довільному відношенні площин електродів. Модель доповнена формулою, яка визначає з точністю до кількох відсотків значення потенціалу зсуву, при якому зонд знаходиться під потенціалом плазми, а вольтамперна характеристика поділяється на перехідну і електронну області. Аналітична залежність потенціалу зсуву від параметрів плазми і відношення площин електродів дозволила формалізувати процедури відновлення і оцінювання достовірності знайдених параметрів плазми з використанням областей їх найбільшого впливу на збираємий зондовий струм. Проведено параметричні дослідження впливу параметрів плазми на зондовий струм в близьких до іоносферних вимірювань умовах. Виконано обґрунтування можливості поділу шуканих параметрів плазми за областями їх найбільшого і найменшого впливу на зондовий струм в діапазоні розглянутих потенціалів зсуву. Сформульовано задачу ідентифікації параметрів плазми на основі порівняння в L_2 теоретичної апроксимації зондового струму і результатів вимірювання ВАХ. Кожному параметру відповідає своя цільова функція, що відрізняється областю визначення і відношенням площин електродів при вимірюванні вольтамперної характеристики. На основі такої постановки оберненої задачі в L_2 отримані оцінки достовірності відновлення параметрів плазми з двохсортними іонами в залежності від похибок моделі і зондових вимірювань. Отримані результати можуть бути використані в діагностиці іоносферної плазми.

Ключові слова: *двохсортні іони плазми, ізольована зондова система з циліндричними електродами, математична модель збирання струму, параметрична ідентифікація, достовірність відновлення параметрів плазми.*

The aim of this work is to estimate the reliability of extracting the plasma electron density and temperature and ionic composition from the current-voltage (I-V) characteristic of an isolated probe system with cylindrical electrodes. An earlier proposed mathematical model of current collection by the probe system at positive bias potentials and an arbitrary ratio of the electrode areas is analyzed. The model is supplemented with a formula that determines, with an accuracy of several percent, the value of the bias potential at which the probe is under the plasma potential and the I-V characteristic splits into a transition and an electronic region. The analytical dependence of the bias potential on the plasma parameters and the ratio of the electrode areas made it possible to formalize the procedures for determining and assessing the reliability of the extracted plasma parameters using the regions of their strongest effect on the collected probe current. Parametric studies of the effect of the plasma parameters on the probe current were carried out for conditions close to measurements in the ionosphere. The paper demonstrates the feasibility of partitioning the sought-for plasma parameters into the regions of their strongest and weakest effect on the probe current in the range of the bias potentials considered. The problem of plasma parameter identification is formulated on the basis of a comparison of the probe current and the measured I-V characteristic in the L_2 theoretical approximation. To each parameter there corresponds an objective function of its own, which differs in the domain of definition and the ratio of the electrode areas used in I-V characteristic measurements. Based on this formulation of the inverse problem in L_2, estimates of the reliability of identification of the parameters of a plasma with two ion species are obtained depending on the errors of the model and probe measurements. The results obtained may be used in ionospheric plasma diagnostics.

Keywords*: two species of plasma ions, isolated probe system with cylindrical electrodes, mathematical model of current collection, parametric identification, reliability of plasma parameter extraction.*

Introduction. Stationary electric Langmuir probes are widely used in diagnostics of space plasma [1, 2]. As a reference electrode for the probe, the body of the spacecraft is usually used – the outer conductive elements of spacecraft, which are not insulated from the plasma. A rather stringent condition $S_s = S_{cp}/S_p \geq 10^4$ for the ratio of the areas of the reference electrode S_{cp} and the probe S_p, along with the strongly rarefied plasma and the current trend towards the use of micro- and nanosatellites, make it difficult to use the measuring circuit of a single Langmuir probe. In such situation, it makes sense to use a measuring probe system that is electrically isolated from the spacecraft body [1].

In [3] we proposed a procedure for interpreting the I-V characteristic of an isolated probe system with composite cylindrical electrodes in a four-component plasma flow using a priori information on the experimental conditions. The procedure is based on the parametric identification of two I-V characteristics of the probe system obtained under the ratio of the electrode areas of S_s <150 and S_s >400. In this case, it was proposed to carry out measurements in the electronic part of I-V characteristic at relatively high bias potentials. However, probe bias potentials of about 10 V are often used in the ionosphere. In this article, we consider the problem of diagnosing a plasma flow with two ion species at lower positive probe bias potentials, and investigated the reliability of the restoration of the electron density and temperature together with the determination of the plasma ionic composition.

Problem formulation. The probe system is isolated from the spacecraft body, it consists of a thin cylindrical measuring electrode (probe) with a base radius r_p and a reference electrode made up of a row of parallel cylinders with base radii r_{cp}. Let the base radii of the electrodes be significantly less than their length, the ends are isolated from the plasma. Let the base radii of the probe and the reference electrode satisfy the conditions:

$$r_p/\lambda_d \le 1,\ r_{cp}/\lambda_d < \xi^* = 3 - 10 \tag{1}$$

where λ_d is the Debye length in unperturbed plasma, ξ^* is the value of r_{cp}/λ_d that limits the applicability of the Langmuir formula for the ion current to a transversely oriented cylinder. With an increase in the flow velocity, ξ^* increases up to 10 in hypersonic flow [4].

Let the electrodes have electrical contact with plasma, the electrostatic and gas-dynamic influence of the electrodes on each other in plasma is small, and there are no emission currents from the surfaces. The probe system is transversely placed in a supersonic plasma flow with mass velocity V. We assume that plasma is four-component (consists of neutrals, positive singly charged ions of two types and electrons), quasi-neutral, the flow around electrodes is collisionless, the influence of the magnetic field on the probe current is insignificant, the velocity distribution of particles of the same type in unperturbed plasma is Maxwellian.

Taking into account the condition of quasineutrality $n_{i,1} + n_{i,2} = n_e$, the ionic composition of the plasma is determined by the parameter $\chi_n = n_{i,1}/n_e$, where $n_{i,1}$, $n_{i,2}$ are, respectively, densities of ions of the species $i,1$ and $i,2$, n_e is the density of electrons. The temperatures of the ions of the species $i,1$ and $i,2$ are assumed to be the same $T_{i,1} = T_{i,2} = T_i$, the masses of the ions $m_{i,1}$, $m_{i,2}$ are known. We assume that $m_{i,1} < m_{i,2}$. The aim is to determine the density n_e and temperature T_e of electrons as well as parameter χ_n by the results of measurements of the I-V characteristic of the considered probe system, i.e. by the dependence of the probe current I_p on the probe's potential with respect to the reference electrode's.

Mathematical model of current collection. A mathematical model of the current collection by an isolated probe system in a plasma with two ion species is developed in [3]. The model is based on the classical Langmuir relations and the results of calculations by Laframboise and works [4, 5] under the assumption that the presence of different types of ions in a supersonic plasma flow does not lead to a significant change in the self-consistent electric field in the vicinity of the cylinder [3, 6]. In dimensionless form, the total current on a cylinder with a potential φ relative to the undisturbed plasma, is estimated by the relations (the electron current is assumed positive):

$$\bar{I}_c(\varphi)=\bar{I}_e(\varphi)-\chi_n\sqrt{\chi_m\,\mu/\beta}\cdot\bar{I}_{i,1}(\varphi)-(1-\chi_n)\sqrt{\mu/\beta}\cdot\bar{I}_{i,2}(\varphi),\quad S_i>\sqrt{\chi_m} \qquad (2)$$

where $\bar{I}_c$, $\bar{I}_e$ are total and electron currents on the cylinder, respectively, normalized by the thermal electron current; $\bar{I}_i$ is the ion current on a cylinder, normalized by the thermal current of ions of the corresponding type, $\chi_m=m_{i2}/m_{i1}$ is the masses ratio of ions of different types, $\mu=m_e/m_{i2}$ is the ratio of the masses of charged particles, $\beta=T_e/T_i$ is the ratio of temperatures of charged particles, $S_i=V/u_{i,2}$ is the ionic velocity ratio for heavier ions of the type $i,2$. The dimensionless potential φ is normalized by kT_e/e, where k is the Boltzmann constant, e is the elementary charge. Here, values with index *i* relate to ions, *i*,1 – to lighter ions of the sort *i*,1, *i*,2 – to heavier ions of the sort *i*,2, *e* – to electrons. Calculation formulas for $\bar{I}_e$, $\bar{I}_i$ are given in [3].

It should be noted that the velocity ratio of lighter ions is $\sqrt{\chi_m}$ times less then S_i. For example, at altitude of about 500 km the velocity ratio for oxygen ions is greater than 7, but for hydrogen ions it's about 2. In this case, condition (1) of applicability of the approximation of the hydrogen ion current to the reference electrode may be violated. In this case, it is necessary to decrease the radius r_{cp}.

Direct problem of probe measurements. The considered isolated probe system is always in a state of equilibrium [1], determined by zero total current of charged particles through the collecting surfaces of the electrodes. The equilibrium potential of the reference electrode φ_{cp}, corresponding to the value of the bias potential φ_{iz}, is determined from the current balance equation in dimensionless form [3]

$$S_s\cdot\bar{I}_c(\varphi_{cp})+\bar{I}_c(\varphi_p)=0 \qquad (3)$$

where $\bar{I}_c(\varphi_{cp})$ is the dimensionless current to the reference electrode, $\bar{I}_c(\varphi_p)$ is the dimensionless current to the probe, $\varphi_p=\varphi_{iz}+\varphi_{cp}$ and is the equilibrium potential of the probe corresponding to the bias potential φ_{iz}.

It was shown in [3] that the probe current $\bar{I}_p(\varphi_{iz})=\bar{I}_c(\varphi_{iz}+\varphi_{cp})$ is a continuous piecewise-analytic function of the bias potential and parameters χ_n, χ_m, μ, β, S_i, S_s. Breaks of smoothness of the function $\bar{I}_p(\varphi_{iz})$ take place at points that separate regions where different calculation formulas used in (2):

$$\varphi_{iz} = -\varphi_{cp}\,, \;\; \varphi_{iz} = -\varphi_{cp} + S_i^{\,2} \big/ (\beta \cdot \chi_m), \;\; \varphi_{iz} = -\varphi_{cp} + S_i^{\,2} \big/ \beta\,. \qquad (4)$$

The first condition determines the boundary between transition and electronic parts of I-V characteristic. The second and third conditions correspond for ions of species *i*,1 and *i*,2 to the separation between ion and transition parts of VAC of the reference electrode, which is always at a negative potential with respect to the undisturbed plasma. Equalities (4) determine for the probe system with area ratio of S_s such surfaces in the space of variables ($\varphi_{iz}, \chi_n, \chi_m, \mu, \beta, S_i$), on which the derivatives of the function $\bar{I}_p(\varphi_{iz}, \chi_n, \chi_m, \mu, \beta, S_i, S_s)$ have discontinuities of the first kind.

Thus, the function $\bar{I}_p(\varphi_{iz}, \chi_n, \chi_m, \mu, \beta, S_i, S_s)$ that determines the I-V characteristic of an isolated probe system and all its derivatives are square-integrable with respect to the bias potential. We find the bias potentials at which the smoothness of the function $\bar{I}_p(\varphi_{iz}, \chi_n, \chi_m, \mu, \beta, S_i, S_s)$ is violated by substituting right-hand side of equality (4) instead of φ_{iz} in (3) and solving the resulting equation with respect to the equilibrium potential φ_{cp}. We obtain the value of φ_{iz} by substituting the found φ_{cp} into the corresponding equality (4).

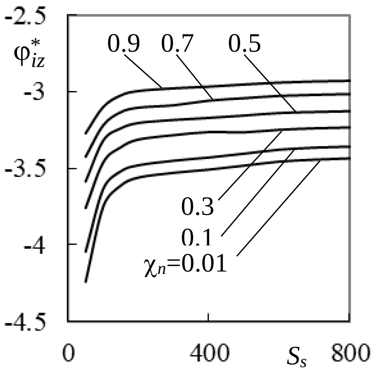


Fig. 1

Let's denote by φ^*_{iz} the probe bias potential and by φ^*_{cp} corresponding equilibrium potential of the reference electrode relative to the plasma, at which the probe potential equals the plasma potential, i.e., the first condition in (4) is satisfied. Fig. 1 shows the dependences of the bias potential φ^*_{iz} on the area ratio S_s at the values of χ_n =0.01, 0.1, 0.3, 0.5, 0.7, 0.9. The calculations were performed for $S_i = 5$, $\mu = 3.43 \cdot 10^{-5}$, $\chi_m = 16$ and $\beta = 1.2$. It appears that over the considered domain of parameters χ_n and S_s, the value of the bias potential changes quite significantly.

For the first condition in (4), which determines the boundary between the transition and electronic regions of the I-V characteristic, the approximation of the equilibrium potential of the reference electrode is found:

$$\varphi^*_{cp} \approx \ln\!\left(2\big/\sqrt{\pi}\cdot\sqrt{\mu/\beta}\right) + \\ + \ln\left(\begin{aligned} &\chi_n \cdot \sqrt{S_i^{\,2} + \chi_m (1/2 - \beta\cdot\varphi_{fl})} + (1-\chi_n)\cdot\sqrt{S_i^{\,2} + 1/2 - \beta\cdot\varphi_{fl}} + \\ &+ \frac{1}{S_s}\cdot\left(\chi_n\cdot\sqrt{S_i^{\,2} + \chi_m/2} + (1-\chi_n)\cdot\sqrt{S_i^{\,2}+1/2} - \frac{1}{2\big/\sqrt{\pi}\cdot\sqrt{\mu/\beta}}\right) \end{aligned} \right)$$

where $\varphi_{fl} \approx \ln\!\left(\sqrt{\mu/\beta}\cdot 2\big/\sqrt{\pi}\cdot S_i\right)$ is the approximate value of the floating potential of a long cylinder transversely placed in plasma flow. For $S_s \in [50{,}1000]$,

$\beta \in [1,1.5]$ and $\chi_n \le 0.9$ the error of the approximate formula does not exceed 1.6% at S_i =7, 2.5% at S_i =5, 3.7% at S_i =3, 5.7% at S_i =2; for $\chi_n \le 0.5$ the error does not exceed 1.4% at S_i =7, 1.7% at S_i =5, 2.8% at S_i =3, 3.5% at S_i =2. As parameters S_s, S_i increase and parameters β, χ_n decrease, the approximate formula becomes more accurate. The value of the bias potential of the probe relative to the reference electrode, at which the first condition from (4) meets, is determined by the relation $\varphi_{iz}^* = -\varphi_{cp}^*$. In what follows, we take the value of the dimensionless bias potential φ_{iz}^* as the boundary separating the transition and electronic parts of the I-V characteristic.

To apply relations (2), (3) in modeling probe measurements in the ionosphere, following [3], we use the geometric parameters of the isolated probe system S_p, S_s and the parameters of the plasma flow $\{n_e, T_e, \chi_n, \chi_m, \mu, \beta, V\}$. Let's select from this set several plasma parameters of our further concern, and denote the selected set as $G = (g_1,.., g_K)$. Here $K \le 7$ is the number of selected parameters. The rest of the plasma parameters are assumed to be given.

In dimensional form, the dependence of the probe current I_p on its potential relative to the unperturbed plasma $U_p = U_{iz} + U_{cp}$ can formally be written as follows:

$$I_p(U_{iz}, S_s, G) = I_{e0}(G) \cdot \bar{I}_c\left(eU_{iz}/kT_e + \varphi_{cp}(eU_{iz}/kT_e, S_s, G), G\right) \qquad (5)$$

where the equilibrium potential $\varphi_{cp}(eU_{iz}/kT_e, S_s, G)$ is to be found as the solution of the current balance equation (3) with substituted parameters (S_s, G).

Analysis of problem parameters. Let us consider the influence of plasma parameters on the probe current $\bar{I}_p$. Numerical studies is carried out for basic values of parameters close to the parameters of the ionosphere at night at an altitude of about 500 km: $n_e = 9 \cdot 10^{10}$ m^{-3}, $T_e = 1200$ K, $\chi_n = 0.1$, $\chi_m = 16$, $\mu = 3.43 \cdot 10^{-5}$, $\beta = 1.2$, $V = 7500$ km/s.

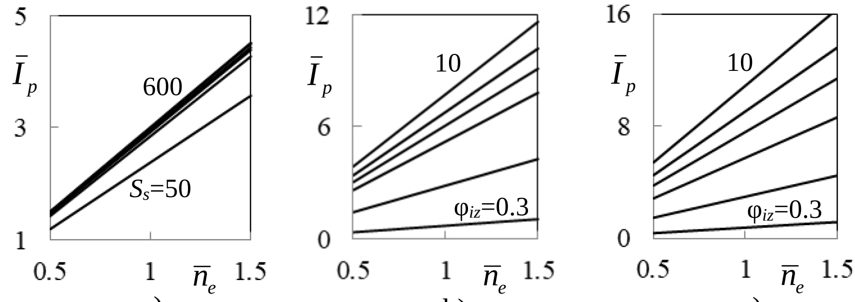
Let us denote by G_0 the vector of the above values of the considered parameters. Changes in parameters is characterized by relative values $\bar{g}_j = g_j / g_{0j}$, where g_j is the current value, g_{0j} is the base value of the parameter, $j = 1..K$.

Fig. 2 – Fig. 5 show the dependence of the probe current $\bar{I}_p$ on the relative values of the main problem parameters $\{n_e, T_e, \chi_n, V\}$: for electrode areas ratio of S_s = 50, 100, 150, 200, 400, 600 at bias potential φ_{iz} = 1 V (a); for potentials φ_{iz} = 0.3, 1, 3, 5, 7, 10 V at S_s =100 (b) and at S_s =400 (c). It is shown the effect of the electron density n_e (Fig. 2), electron temperature T_e (Fig. 3), parameter χ_n (Fig. 4), mass flow velocity V (Fig. 5) on the probe current. Since the parameter T_i is assumed to hold when T_e varies, the parameter β also vary synchronously with T_e. An increase in the bias potential φ_{iz} and electrode areas ratio S_s with fixed other parameters

leads to an increase in the probe current. Therefore, $\bar{I}_p$ curves corresponding to the series of parameters S_s = 50, 100, 150, 200, 400, 600 (a) and φ_{iz} = 0.3, 1, 3, 5, 7, 10 V (b, c) do not intersect and arranged from bottom to top.

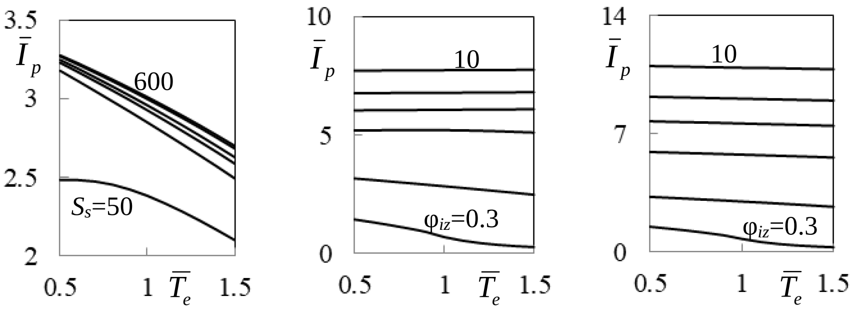
The mathematical model (2) – (3) determines the linear dependence of the probe current on the electron density n_e. Therefore, in Fig. 2 dependence of $\bar{I}_p$ on n_e represent straight lines. Thus, the function $I_p(U_{iz}, S_s, G)$ and its derivatives with respect to other parameters depend linearly on n_e. With an increase in the bias potential φ_{iz} and area ratio S_s, the influence of the electron density n_e on the collected probe current increases.

Analysis of the results presented in Fig. 3, a) – Fig. 5, a) shows that with an increase in the area ratio S_s, the dependence of the probe current on the parameters T_e, χ_n and V approaches became almost linear. The results presented in Fig. 3, b), c) – Fig. 5 b), c) show that with an increase in the bias potential φ_{iz}, the dependence of the probe current on T_e and V also approaches to linear, but the dependence on χ_n, on the contrary, tend to be linear for smaller φ_{iz}.



a) b) c)

Fig. 2



a) b) c)

Fig. 3

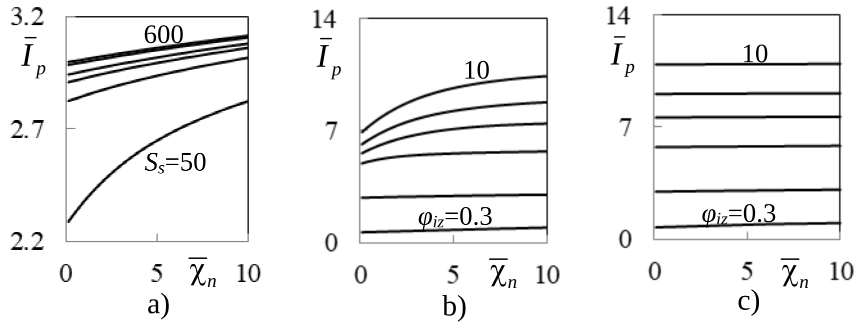


Fig. 4

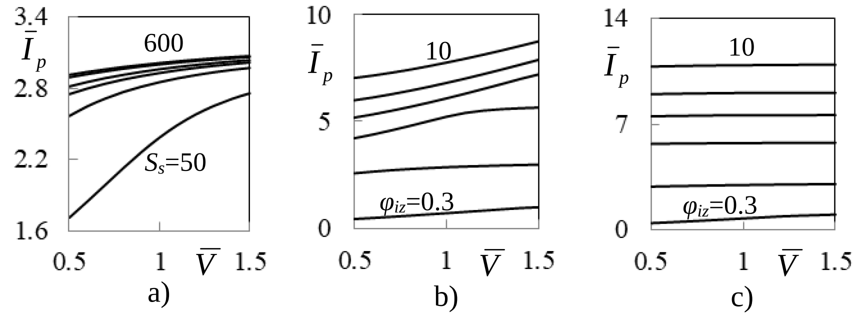


Fig. 5

Shown in Fig. 3 dependence of the probe current on the electron temperature T_e at $\varphi_{iz} \geq 3$ and $S_s \geq 100$ can be assumed to be linear in the considered range of parameter T_e. In this case, the effect of T_e on the collected probe current, by the Langmuir probe theory, is stronger in the transition region of the I-V characteristic at $\varphi_{iz} < 1$. At $\varphi_{iz} \geq 3$ and $S_s \geq 100$, the effect of T_e on the collected probe current is insignificant.

Shown in Fig. 4, the dependence of the probe current on χ_n covers the entire permissible range of parameter variation, from 0 to 1. As the area ratio S_s increases, the effect of χ_n on the collected probe current decreases. The strongest influence of χ_n on the probe current is observed in the electronic part of the I-V characteristic at $\varphi_{iz} > 3$ and $S_s < 150$. In the transition part of the I-V characteristic and at large $S_s > 400$, the effect of the parameter χ_n on the collected probe current is insignificant.

The flow velocity V is known to be the determining factor for the ion current to the electrode in the transition region of the I-V characteristic. At $\varphi_{iz} < 1$, the nonlinear dependence of the probe current on V in Fig. 5 is observed for all considered values of S_s. At $\varphi_{iz} > 3$ and $S_s > 400$, the influence of V on the probe current is negligible.

For the function $I_p(U_{iz}, S_s, G)$ in the vicinity of the base point G_0 we define dimensionless sensitivity functions [3]:

$$\bar{I}_{p,g_j}(U_{iz},S_s,G_0)=\frac{g_{0j}}{I_{e0}(G_0)}\frac{\partial I_p(U_{iz},S_s,G_0)}{\partial g_j}, \quad j=1..K .$$

Using the Taylor formula of the 1st order, for small changes in the parameters $g_j = g_{0j}\left(1+\delta_{g_j}\right)$ in the vicinity of the point G_0, we obtain

$$\bar{I}_p(U_{iz},S_s,G)-\bar{I}_p(U_{iz},S_s,G_0)=\sum_{j=1}^{K}\delta_{g_j}\,\bar{I}_{p,g_j}(U_{iz},S_s,G_0)+\frac{\Delta_I(U_{iz},S_s,G_0,\delta G)}{I_{e0}(G_0)} \quad (6)$$

where $G=G_0\circ(1+\delta G)$ is the vector of varied parameters; $\delta G=(\delta g_1,..,\delta g_K)$ is the vector of relative variations of parameters; $\Delta_I(U_{iz},S_s,G_0,\delta G)$ is the remainder in the Lagrange form, calculated through the second derivatives of the function $I_p(U_{iz},S_s,G)$ at some point $G=G_0\circ(1+\theta\delta G)$ in the set of parameter values, $0<\theta<1$, $\mathrm{a}\circ\mathrm{b}$ is the element-wise product of vectors a and b.

As mentioned above, the function $\bar{I}_p(\varphi_{iz},\chi_n,\chi_m,\mu,\beta,S_i,S_s)$, which determines the dimensionless I-V characteristic of the probe system, and all its derivatives are square-integrable with respect to the bias potential φ_{iz}. Consequently, function (5), which determines the dimensional probe current, and all its derivatives by parameters $\{n_e,T_e,\chi_n,\chi_m,\mu,\beta,V\}$ are square-integrable with respect to the potential U_{iz}, and for them the norm in L_2 on the interval $[U_0,U_1]$ can be determined:

$$\|I(U)\|_{[U_0,U_1]}=\sqrt{\frac{1}{|U_1-U_0|}\int_{U_0}^{U_1}|I(U)|^2\,dU} .$$

For simplicity, parameter S_s in the notation of functions and the interval in the notation of the norm are further omitted.
Using the Minkowski inequality [7], from (6) we obtain:

$$\left\|\bar{I}_p(U_{iz},G)-\bar{I}_p(U_{iz},G_r)\right\|\le\sum_{j=1}^{K}\left|\delta_{g_j}\right|\cdot\left\|\bar{I}_{p,g_j}(U_{iz},G_r)\right\|+\frac{\left\|\Delta_I(U_{iz},G_r,\delta G)\right\|}{\left|I_{e0}(G_r)\right|}.$$

The residual term, normalized by the thermal electron current, is estimated by the second derivatives of the function $I_p(U_{iz},S_s,G)$ on the set of the considered values of parameters $G=G_r\circ(1+\delta G)$. It's possible to avoid the calculation of the second derivatives of the probe current in the following way. Using formula (6) and the properties of the norm, we can write

$$\left\|\frac{\Delta_I(U_{iz},G_r,\delta G)}{I_{e0}(G_r)}\right\|\le\max_{\delta G^{\min}\le\delta G\le\delta G^{\max}}\left\|\begin{gathered}\left[\bar{I}_p(U_{iz},G_r\circ(1+\delta G))-\bar{I}_p(U_{iz},G_r)\right]-\\-\sum_{j=1}^{K}\delta_{g_j}\,\bar{I}_{p,g_j}(U_{iz},G_r)\end{gathered}\right\|.$$

Taking into account the monotonic dependence of the probe current on the plasma parameters, the estimate of the norm on the right-hand side can be found by a simple checking through the extremal values of the parameter range $\delta G^{\min}\le\delta G\le\delta G^{\max}$, where $\delta G^{\min}=\left(\delta g_1^{\min},..,\delta g_K^{\min}\right)$ and

$\delta G^{\max} = \left(\delta g_1^{\max},..,\delta g_K^{\max}\right)$ are, respectively, the vectors of the smallest and largest admissible values of the parameters [3, 5].

Fig. 6 shows the dependences of the sensitivity functions norm $\left\|\bar{I}_{p,g_j}(U_{iz},S_s,G_r)\right\|$ on the area ratio S_s in the transition region (a) in the bias potential range [0, 0.4 V] and in the electron saturation region (b) of I-V characteristic in the bias potential range [0.4 V, 12 V]. The bold solid curve in Fig. 6 corresponds to the function $\left\|\bar{I}_{p,n_e}\right\|$ of sensitivity to parameter n_e, the thin solid curve - function $\left\|\bar{I}_{p,T_e}\right\|$ of sensitivity to parameter T_e, dashed curve - function $\left\|\bar{I}_{p,\chi_n}\right\|$ of sensitivity to parameter χ_n, dotted curve - function $\left\|\bar{I}_{p,V}\right\|$ of sensitivity to parameter V.

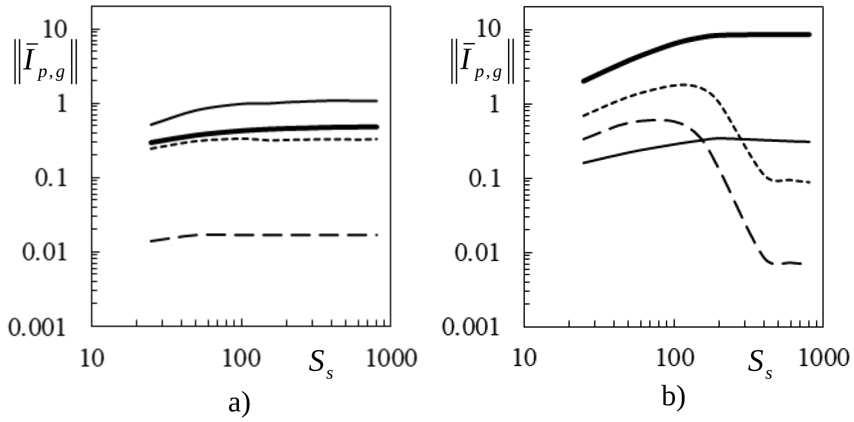


Fig. 6

In the transition region, the norm of the sensitivity function to T_e is almost twice the norm of the sensitivity function to n_e for all values of S_s. In the transition region and in the electron saturation region at S_s >400, the norm of the function of sensitivity to χ_n is very small. In this case, the norm of the function of sensitivity to velocity V exceeds that to χ_n for all values of S_s.

Thus, results of parametric studies at positive bias potentials from 0 to 12 V and more, shown in Fig. 2 – 6 and in [3], prove that:

– the regions on the I-V characteristic where strong influence of n_e and T_e on the current occur, are different for all considered values of S_s: it is electron saturation region for n_e and transition region for T_e, as in the classical theory of a single Langmuir probe;

– the strongest influence of the parameter χ_n is observed in the electron saturation part of the I-V characteristic at S_s <150, and at S_s >400 the effect of χ_n on the collected probe current is insignificant;

– the region of influence of the parameter V coincides with that of χ_n for all considered S_s.

Inverse problem. It was shown in [3] that, in contrast to a single Langmuir probe, the electronic part of the I-V characteristic of an isolated probe system depends on the ion flow velocity V. This makes it possible to interpret probe measurements using only positive bias potentials (transition and electron saturation parts of the I-V characteristic), at which the probe current significantly exceeds the one collected in the ion region of I-V characteristic.

According to the problem formulation, the ion masses (i.e. parameters χ_m, μ) are known. Following [3], we fix the value of the parameter β based on the a priori information about the admissible values of the degree of plasma non-isothermality. In addition to unknown parameters $\{n_e, T_e, \chi_n\}$, we also consider the mass velocity of plasma flow V, assuming it is given with certain accuracy δ_V.

Let us consider a probe system with switchable reference electrodes, which is capable to measure the I-V characteristic at various values of the area ratio S_s [3]. Let S_s^* and S_s^{**} be different area ratios ($S_s^* < S_s^{**}$). We have experimentally obtained I-V characteristic for both measuring systems, $I_{ms}\left(U_{iz}, S_s^*\right)$ and $I_{ms}\left(U_{iz}, S_s^{**}\right)$. Taking into account the above considerations about the influence of problem parameters on the probe current, the problem of reconstructing unknown plasma parameters $\{n_e, T_e, \chi_n\}$ at $S_s^* < 150$ and $S_s^{**} > 400$ becomes:

$$n_e^*:\ F\left(S_s^{**}, G^*, \Omega_2\right) = \min_{n_e^{\min} \le n_e \le n_e^{\max}} F\left(S_s^{**}, G, \Omega_2\right), \tag{7}$$

$$T_e^*:\ F\left(S_s^{**}, G^*, \Omega_1\right) = \min_{T_e^{\min} \le T_e \le T_e^{\max}} F\left(S_s^{**}, G, \Omega_1\right), \tag{8}$$

$$\chi_n^*:\ F\left(S_s^{*}, G^*, \Omega_2\right) = \min_{\chi_n^{\min} \le \chi_n \le \chi_n^{\max}} F\left(S_s^{*}, G, \Omega_2\right), \tag{9}$$

where $F(S_s, G, \Omega_{iz}) = \left\| I_p(U_{iz}, S_s, G) - I_{ms}(U_{iz}, S_s) \right\|_{\Omega_{iz}}$ is the norm in L_2 on the interval of bias potential Ω_{iz} of the difference between the theoretical $I_p(U_{iz}, S_s, G)$ and experimental $I_{ms}(U_{iz}, S_s)$ I-V characteristics, corresponding to the areas ratio of S_s; $\Omega_1 = \left[0, U_{iz}^*\right]$ corresponds to the transition region on I-V characteristic, $\Omega_2 = \left[U_{iz}^*, U_{iz}^{\max}\right]$ – to the electron region on I-V characteristic; U_{iz}^* is the value of the bias potential that corresponds to the zero potential of the probe relative to the plasma potential; $U_{iz}^{\max}$ is the largest value of the bias potential. The superscripts "min" and "max" indicate the lowest and highest parameter values, respectively. For the parameter χ_n, its entire range from $\chi_n^{\min} = 0$ to $\chi_n^{\max} = 1$ is used. When calculating the functional $F(S_s, G, \Omega_{iz})$, a piecewise linear interpolation is applied to the experimentally obtained discrete functions $I_{ms}(U_{iz}, S_s)$.

Problem separation into a sequence of variation problems (7) – (9) is done in order to obtain adequate estimates of the reliability of parameters determination.

Determination of parameters $\{n_e, T_e, \chi_n\}$ is performed by sequential solving variation problems (7) – (9), initial guess is made using a priori information about the investigated plasma flow [3, 5, 6]. Next variation problem uses solution of previous problem as an initial guess. This approach, similar to the classical method of processing the I-V characteristic of a single Langmuir probe, allows using for every unknown parameter only parts of I-V characteristic where its influence on collected current is strong.

Since the domain of definition of the functional (norm) can change with changing parameters, solving each problem is carried out on a fixed domain, and U_{iz}^* is calculated only once before solving each variation problem.

The performed calculations proved that with an adequate choice of initial guess, the first iteration of solving each variation problem (7) – (9) returns decent for practical use solution. Usually, 2-3 more iterations is enough to get more accurate results, when necessary.

Let us denote by $I_r(U_{iz})$ real current that probe naturally collects, $I_{ms}(U_{iz})$ is measured probe current, $G_r = (g_{r1},..,g_{rK})$ is real values of plasma parameters, $G^* = (g_1^*,..,g_K^*)$ is the solution of the inverse problem (7) – (9). Let $\delta G = (\delta g_1,..,\delta g_K)$ be vector of relative deviations of G_r from G^*, such that $G_r = G^* \circ (1+\delta G)$. The function $I_r(U_{iz})$ is assumed to be square integrable. Vectors G_r and δG are unknown.

Considering the identity

$$\begin{aligned}&\bar{I}_p(U_{iz},G_r) - \bar{I}_p(U_{iz},G^*) = \\ &= \left[\bar{I}_p(U_{iz},G_r) - \bar{I}_r(U_{iz})\right] + \left[\bar{I}_r(U_{iz}) - \bar{I}_{ms}(U_{iz})\right] + \left[\bar{I}_{ms}(U_{iz}) - \bar{I}_p(U_{iz},G^*)\right]\end{aligned}$$

and using the Minkowski inequality for each norm corresponding to variation problems (7) – (9), we obtain the estimate:

$$\begin{aligned}&\left\|\bar{I}_p(U_{iz},G_r) - \bar{I}_p(U_{iz},G^*)\right\| \le \left\|\bar{I}_r(U_{iz}) - \bar{I}_p(U_{iz},G_r)\right\| + \left\|\bar{I}_r(U_{iz}) - \bar{I}_{ms}(U_{iz})\right\| + \\ &+ \left\|\bar{I}_{ms}(U_{iz}) - \bar{I}_p(U_{iz},G^*)\right\| \le \varepsilon_M + \varepsilon_{ms} + \varepsilon_F\end{aligned}, \qquad (10)$$

where ε_M is the error of the mathematical model (2) - (4), it limits the first term of right-hand side; ε_{ms} is the error in electric current measurement, it limits the second term; ε_F is minimum of the functional of the problem (7) – (9), it limits the third term. In general, ε_M, ε_{ms} and ε_F depend on S_s and the interval at which the norm is calculated.

Assume that a majorant M_2 exists for the normalized residual term in (6), so that $\left\|\Delta_I(U_{iz},S_s,G^*,\delta G)\right\| / \left|I_{e0}(G^*)\right| \le M_2$ for $\|\delta G\|_C \le \delta$. Let us estimate the error of parameter determination g_i, $i \in [1,..,K]$. Solving (6) with respect to the term containing g_i, using the Minkowski inequality and the positivity of the norm of the sensitivity functions, taking into account (10), we obtain

$$\left|\delta_{g_i}\right| \le \frac{\varepsilon_\Sigma}{\left\|\bar{I}_{p,g_i}\left(U_{iz},S_s,G^*\right)\right\|} + \sum_{\substack{j=1\\ j\ne i}}^{K}\left|\delta_{g_j}\right| \cdot \frac{\left\|\bar{I}_{p,g_j}\left(U_{iz},S_s,G^*\right)\right\|}{\left\|\bar{I}_{p,g_i}\left(U_{iz},S_s,G^*\right)\right\|} \tag{11}$$

where $\varepsilon_\Sigma = \varepsilon_M + \varepsilon_{ms} + \varepsilon_F + M_2$ is the total error of the mathematical model, probe measurements and problem linearization by parameters. Terms in (11), including the error ε_Σ, are estimated by the norms in accordance with the variation problem from (7) – (9) for parameter g_i. Note that the norms and the error ε_Σ in (11) are written for the corresponding absolute values, normalized by the thermal electron current, while δ_g is the relative errors of the parameters.

Estimates (11) are also valid for the discrete quadratic norm on grid-defined functions $\bar{I}_r(U_{iz})$, $\bar{I}_p(U_{iz},G_r)$, $\bar{I}_p(U_{iz},G)$.

Considering V as a known parameter with uncertainty δ_V, from (11) we obtain three inequalities that estimate the uncertainty of each parameter values from the set $\{n_e, T_e, \chi_n\}$ through that of the remaining two. Solving the resulting system of linear inequalities, we estimate the reliability of the determination of unknown values $\{n_e, T_e, \chi_n\}$ corresponding to the parameters $\left(S_s, G^*\right)$.

Fig. 7 shows the obtained dependences of the uncertainty in the parameters determination δ_g on ε_Σ. Solid curve corresponds to the uncertainty δ_{n_e} of the n_e determination, dashed curve – to the uncertainty δ_{T_e} of the T_e determination, dotted curve – to the uncertainty δ_{χ_n} of the χ_n determination, dots – to the uncertainty δ_{χ_n} for wider potential range $U_{iz}^{\max} = 100$ V in the formulation of variation problems (7), (9). The calculations is carried out for electrodes areas ratio of S_s^*=50 and S_s^{**}=600, the uncertainty in the flow velocity around the electrodes of $\delta_V = 0.01$.

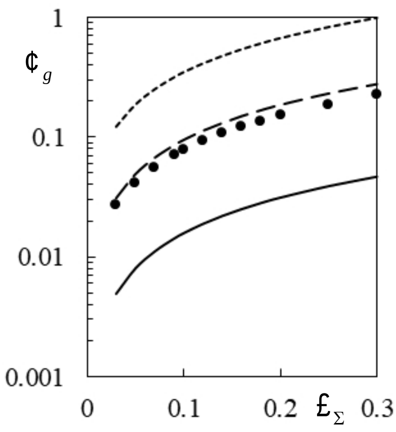


Fig. 7

The obtained estimates showed that the proposed procedure for diagnosing plasma parameters by the I-V characteristic of an isolated probe system in terms of the reliability of electrons density and temperature determination isn’t worse than measurements by a single Langmuir probe at all considered intervals of the bias potential from 0 to 10 V and more. The reliability of determining the mass composition (parameter χ_n) of plasma with two ions species is not sufficient for measurements in the ionosphere using bias potential range from 0 to 12 V. For wider (tens of volts) interval of the bias potential, the reliability of the electron density and χ_n determination increases, while the reliability of the electron temperature, which is determined by the transition part of the I-V characteristic, determination increases insignificantly. With a range from 0 to 100 V, as shown in Fig. 7, the reliability of the parameter χ_n determination is not worse than that of the electron temperature.

Conclusions. A procedure for determination of parameters of the low-temperature collisionless plasma flow with two ions species by the I-V characteristic of an isolated probe system with cylindrical electrodes is developed. The analysis of the previously developed mathematical model of current collection by the probe system at positive bias potentials and an arbitrary electrode areas ratio is carried out. An approximate formula is obtained to calculate for the given plasma flow and probe system such value of bias potential that separates the theoretical I-V characteristic into the transition and electronic parts. Then, each individual plasma parameter is determined considering the ranges of their strong and weak influence on the collected current. For conditions close to ionospheric probe measurements, it is estimated how uncertainty in plasma parameters affects the collected probe current.

The problem of identifying the density, temperature of electrons and the ionic composition of plasma is formulated as a variation problem for which the objective function is the difference in L_2 between theoretical and measured I-V characteristics. Each parameter has its own objective function, with its own domain of definition and settings of measuring probe system. On the basis of this formulation of the inverse problem, estimates of the reliability of determining the plasma parameters are obtained depending on the errors of the mathematical model and probe measurements. It is shown that to reliably determine the mass composition of plasma with two-sorted ions, wide interval of bias potential up to tens of volts is required in probe measurements.

The results obtained can be used in diagnostics of ionospheric plasma.

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