UDC 537.87; 621.372 https://doi.org/10.15407/itm2021.01.084

P. I. ZABOLOTNYI

MODEL OF H-POLARIZED WAVE PROPAGATION IN THE MULTILAYER DIELECTRIC STRUCTURE

Institute of Technical Mechanics
of the National Academy of Sciences of Ukraine and the State Space Agency of Ukraine
15 Leshko-Popelya St., Dnipro 49005, Ukraine; e-mail: mwave@ukr.net

В роботі розглядається питання визначення діелектричної проникності багатошарових діелектричних структур. Відзначено, що одним з найбільш розповсюджених методів визначення діелектричної проникності багатошарових структур є вимір коефіцієнтів відбиття електромагнітних хвиль інтерференційними методами. У загальному випадку при проведенні вимірів інтерференційними методами, одному виміряному значенню коефіцієнта відбиття може відповідати безліч значень діелектричної проникності. Ця невизначеність може бути усунена, якщо є можливість попереднього визначення впливу на коефіцієнти відбиття різних параметрів зондуючих електромагнітних хвиль. Зокрема є важливим отримання попередньої оцінки впливу кутів падіння та поляризації на діапазон зміни коефіцієнтів відбиття при зміні одного з параметрів структур. Це дозволяє отримати попередню оцінку меж діапазону зміни коефіцієнта відбиття при зміні параметру, який досліджується.

У даній роботі розглянуто випадок падіння на багатошарову діелектричну структуру плоскої електромагнітної хвилі, у якій магнітне поле перпендикулярно площині падіння, тобто з Н-поляризацією. Мета роботи – розробка моделі поширення Н-поляризованої електромагнітної хвилі скрізь багатошарову діелектричну структуру при довільному куті падіння і визначення межі діапазону зміни коефіцієнта відбиття при зміні діелектричної проникності її шарів. Розроблено модель розповсюдження Н-поляризованої електромагнітної хвилі в двошаровій діелектричній структурі, яку розташовано на металевій основі з ідеальною провідністю. Кут падіння електромагнітної хвилі на неї з повітря є довільним. Запропонована модель дозволяє оцінювати коефіцієнт відбиття електромагнітної хвилі по параметрам структури і куту падіння хвилі. Модель дає можливість отримати аналітичну оцінку межі діапазону зміни коефіцієнта відбиття при зміні діелектричної проникності і товщини кожного з шарів діелектричної структури. З використанням розробленої моделі було отримано залежності модуля коефіцієнта відбиття від кута падіння та діелектричної проникності другого шару.

Ключові слова: *Н-поляризація, діелектрична проникність, коефіцієнт відбиття, багатошарові діелектричні структури.*

This paper addresses the determination of the dielectric constant of multilayer dielectric structures. One of the most-used methods for determining the dielectric constant of multilayer structures is reflection coefficient measurement by interferometry. In the general case, in interferometry measurements to one measured value of the reflection coefficient there may correspond an infinity of dielectric constants. This ambiguity may be resolved by first determining the effect of different parameters of the probing electromagnetic wave on the reflection coefficient. In particular, it is important to have a preliminary estimate of the effect of the incidence angle and the polarization on the range of variation of the reflection coefficient with the variation of one of the structure parameters. This allows one to estimate the boundaries of the range of variation of the reflection coefficient with the variation of the parameter under study.

This paper considers the case where a plane H-polarized electromagnetic wave, i.e. a wave whose magnetic field is perpendicular to the incidence plane, is incident on a multilayer dielectric structure. The aim of this work is to develop a model of the propagation of an H-polarized electromagnetic wave through a multilayer dielectric structure at an arbitrary incidence angle and to determine the range of variation of the reflection coefficient with the variation of the dielectric constants of the layers. The paper presents a model of the propagation of an H-polarized electromagnetic wave in a two-layer dielectric structure. A metal base, which is an ideal conductor, underlies the structure. The electromagnetic wave is incident from the air at an arbitrary incidence angle. The model allows one to estimate the reflection coefficient of the structure as a function of its parameters and the incidence angle. The model also makes it possible to analytically estimate the range of variation of the reflection coefficient with the variation of the dielectric constant and the thickness of each layer of the structure. Using the model, the magnitude of the reflection coefficient was determined as a function of the incidence angle and the dielectric constant of the second layer.

Keywords: *H-polarization, dielectric constant, reflection coefficient, multilayer dielectric structures.*

To study the properties or control parameters of various materials commonly used methods to measure dielectric constant using electromagnetic waves at microwave frequencies.

The great scientific and practical interest in radar, radio communications, antennas technology, the development of new coatings and many other applications is needs to create electromagnetic models of objects that are randomly located in space in relation to the incident electromagnetic waves on their surface [1 – 6].

One of the most common subjects for the research of their dielectric properties are arbitrarily arranged multilayer media with flat layers.

An important part of this researchers is to measure the dielectric constant of individual layers and effectively dielectric constant of the object in general. One of the methods for determining the dielectric constant is the measurement of the reflection coefficient by interference methods.

However, in the general case, one measured value of the reflection coefficient can correspondence to many values of the dielectric constant [7]. This uncertainty can be eliminated by determining the reflection coefficient at different parameters of the probing electromagnetic radiation, the different polarization of the waves and the angles of incidence.

When studying the effect of the polarization effect of a plane wave to the reflection coefficient, it is necessary to determine the plane of incidence of the wave. This plane usually is defined as the plane that passes through the direction of wave propagation and the normal to the interface.

Usually two separate cases are considered [7]:

- the wave has a polarization normal to the plane of incidence;
- the wave is polarized in the plane of incidence (the plane of polarization and the plane of incidence coincide).

In this paper, we consider the case when a plane electromagnetic wave falls in a multilayer structure, in which the magnetic field is perpendicular to the plane of incidence of H-polarization.

The goal of the work – development the model of H-polarized wave propagation in the multilayer dielectric structure at an arbitrary angle of incidence and determination of the boundary of the range of change of the reflection coefficient when the dielectric constant of its layers changes.

Consider the model of the layered structure, which is presented in fig. 1 and which consists of two dielectric layers, on the surface of the metal.

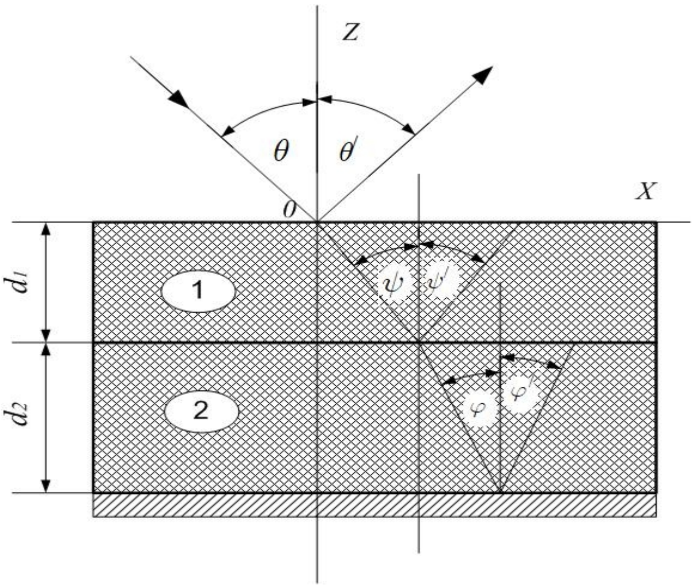

Fig. 1

The first dielectric layer has a thickness d_1 and a dielectric constant e_1. The second dielectric layer has a thickness d_2 and a dielectric constant e_2.

An electromagnetic wave with H-polarization falls on the surface of the multilayer structure from the air. We assume that the dielectric constant of air e_0 equal to the dielectric constant of the vacuum. Let the normal to the wavefront of the incident waveform an angle ¯ with a perpendicular to the interface.

We introduce a coordinate system that begins at the boundary between air and the first dielectric. The *OZ* axis is perpendicular to the interface and is directed to the dielectric.

The *OX* axis is in the plane of incidence. The *OY* axis is directed along the vector of the magnetic field of the incident wave.

In this case, the vector of the magnetic field of the incident wave has only one component, which is described by the expression

$$H_y = H_m e^{-gx} \tag{1}$$

where H_m – amplitude multiplier; γ – propagation constant electromagnetic wave in the medium;ξ – the distance from the origin to the wave surface.

The constant propagation γ can be expressed through the dielectric constant of the medium ε and its magnetic permeability μ [8]:

$$g = jw\sqrt{em},$$

where j – imaginary unit; μ – circular frequency.

As in most cases, the propagation of electromagnetic waves in dielectrics must take into account its loss, dielectric constant of the medium conveniently presented as

$$e = e_0 e'(1 - j \cdot \mathrm{tg} d)$$

where e' – relative dielectric constant of the medium; $\mathrm{tg} d$ – dissipation factor.

It is known [3], that any radius vector that is drawn from the beginning of the Cartesian coordinate system to an arbitrary point on the wave surface can be defined by the expression:

$$\vec{r} = \vec{i}_x x + \vec{i}_y y + \vec{i}_z z$$

where $\vec{i}_x, \vec{i}_y, \vec{i}_z$ – unit vectors of the corresponding coordinate axes.

Accordingly, the projection of this vector in the direction of the normal to the wave surface has the form

$$x = \vec{r} \cdot \vec{n} = x\cos(\vec{n}, \vec{i}_x) + y\cos(\vec{n}, \vec{i}_y) + z\cos(\vec{n}, \vec{i}_z)$$

where $\vec{n}$ – unit normal vector to the wave surface.

Given this, expression (1) can be written as:

$$H_y = H_m e^{-g[x\cos(\vec{n}, \vec{i}_x) + y\cos(\vec{n}, \vec{i}_y) + z\cos(\vec{n}, \vec{i}_z)]}. \tag{2}$$

For H-polarization of an electromagnetic wave, the normal to the wavefront of the incident wave lies in the plane of incidence *XOZ* and forms an angle ¯ with the perpendicular to the plane between the layers. This allows writing the trigonometric functions in the exponent (2) as follows:

$$\cos(\vec{n},\vec{i}_x) = \sin(\theta),\ \cos(\vec{n},\vec{i}_y) = 0,\ \cos(\vec{n},\vec{i}_z) = \cos(\theta).$$

Accordingly, the expression for the Y-component of the magnetic field of the incident wave in the air might be written as

$$H_{y0}^{+} = H_0^{+} e^{-\gamma_0(x\sin\theta + z\cos\theta)} \tag{3}$$

where H_0^{+} – the amplitude of the magnetic field of the incident wave in the air; γ_0 – propagation constant electromagnetic wave in the air.

Similarly, might be present expressions for the Y-component of the magnetic field of the reflected wave in air

$$H_{y0}^{-} = H_0^{-} e^{-\gamma_0(x\sin\theta' - z\cos\theta')},$$

where H_0^{-} – the amplitude of the magnetic field of the reflected wave in air; θ' – the angle of the reflection wave in the air.

The resulting magnetic field in the air is the sum of the magnetic fields of the incident and reflected waves

$$H_0 = H_0^{+} e^{-\gamma_0(x\sin\theta + z\cos\theta)} + H_0^{-} e^{-\gamma_0(x\sin\theta' - z\cos\theta')}.$$

Repeating the algorithm of the above transformations, might be write the expressions for the resulting magnetic field of the wave in the first and second layers of dielectrics:

$$H_1 = H_1^{+} e^{-\gamma_1(x\sin\psi + z\cos\psi)} + H_1^{-} e^{-\gamma_1(x\sin\psi' - z\cos\psi')},$$

$$H_2 = H_2^{+} e^{-\gamma_2(x\sin\psi + z\cos\psi)} + H_2^{-} e^{-\gamma_2(x\sin\psi' - z\cos\psi')}$$

where H_1, H_1^{+}, H_1^{-} – the amplitudes of the total magnetic field value, magnetic field of the incident wave and magnetic field of the reflected wave in the first dielectric; H_2, H_2^{+}, H_2^{-} – the amplitudes of the total magnetic field value, magnetic field of the incident wave and magnetic field of the reflected wave in the second; ψ – the angle between the normal to the wavefront of the incident wave at the boundary of the first layer and the normal to the plane of the interface between the first and second layers ($z = d_1$); ψ' – the angle of the reflection wave in the first layer; φ – the angle between the normal to the wavefront of the incident wave at the boundary of the second layer and the normal to the plane of the interface between the second layer and metal plate ($z = d_1 + d_2$); φ' – the angle of the reflection wave in the second layer.

To obtain the algebraic equations for the unknown amplitudes H_n^{+} and H_n^{-}, $n = 0,1,2$ we will use the boundary conditions for the tangential components of the magnetic and electric fields on the surfaces of the boundaries of the considered media.

The condition of equality of magnetic tangential components in the plane $z = 0$ can be written in the form

$$H_0^+ e^{-\gamma_0 x \sin\theta} + H_0^- e^{-\gamma_0 x \sin\theta'} = H_1^+ e^{-\gamma_1 x \sin\psi} + H_1^- e^{-\gamma_1 x \sin\psi'} . \qquad (4)$$

Equation (4) must be true in any coordinates *x*.

This can be done only if the exponential factors containing coordinates x, equal.

This leads to the equations:

$$\gamma_0 \sin\theta = \gamma_0 \sin\theta' = \gamma_1 \sin\psi = \gamma_1 \sin\psi'$$

where it follows

$$\sin\theta = \sin\theta', \ \sin\psi = \sin\psi \ , \ \gamma_0 \sin\theta = \gamma_1 \sin\psi . \qquad (5)$$

Accordingly, when $z = 0$, the boundary conditions for magnetic fields take the form:

$$H_0^+ + H_0^- = H_1^+ + H_1^- . \qquad (6)$$

Expressions for the components of the electric field can be obtained from Maxwell's equations in differential form [3]:

In the future we will be interested only in the tangential component of the electric field E_x, so we will present it through the tangential component of the magnetic field H_y:

$$E_x = -\frac{1}{j\omega\varepsilon}\frac{\partial H_y}{\partial z} . \qquad (7)$$

Taking into account (7), we can write expressions for the resulting electric fields in air E_0, in the first E_1 and in the second E_2 layers of dielectrics, accordingly:

$$E_0 = \frac{\gamma_0 \cos\theta}{j\omega\varepsilon_0}\left\{ H_0^+ e^{-\gamma_0 (x\sin\theta + z\cos\theta)} - H_0^- e^{-\gamma_0 (x\sin\theta - z\cos\theta)} \right\},$$

$$E_1 = \frac{\gamma_1 \cos\psi}{j\omega\varepsilon_1}\left\{ H_1^+ e^{-\gamma_1 (x\sin\psi + z\cos\psi)} - H_1^- e^{-\gamma_1 (x\sin\psi - z\cos\psi)} \right\},$$

$$E_2 = \frac{\gamma_2 \cos\varphi}{j\omega\varepsilon_2}\left\{ H_2^+ e^{-\gamma_2 (x\sin\varphi + z\cos\varphi)} - H_2^- e^{-\gamma_2 (x\sin\varphi - z\cos\varphi)} \right\}.$$

It should be noted, that since the third layer in this structure is a metal with ideal conductivity, the electric field E_3 in it is zero

$$E_3 = 0 .$$

Therefore, the boundary conditions $E_2 = E_3$ in the plane $z = d_1 + d_2$ can be reduced to the equation

$$H_2^+ e^{-\gamma_2(d_1+d_2)\cos\varphi} - H_2^- e^{\gamma_2(d_1+d_2)\cos\varphi} = 0\,. \tag{8}$$

From equation (8) for the reflection coefficient at the boundary between the second dielectric layer and the metal, we have

$$R_2 = \frac{H_2^-}{H_2^+} = e^{-2\gamma_2(d_1+d_2)\cos\varphi}\,. \tag{9}$$

For a plane with a coordinate $z = d_1$ of the equality of the tangential components of the magnetic field $H_1 = H_2$ and the electric field $E_1 = E_2$, the following pair of equations can be obtained

$$H_1^+ e^{-\gamma_1 d_1\cos\psi} + H_1^- e^{\gamma_1 d_1\cos\psi} = H_2^+ e^{-\gamma_2 d_1\cos\varphi} + H_2^- e^{\gamma_2 d_1\cos\varphi}\,, \tag{10}$$

$$\frac{\gamma_1\cos\psi}{\varepsilon_1}\left[H_1^+ e^{-\gamma_1 d_1\cos\psi} - H_1^- e^{\gamma_1 d_1\cos\psi}\right] = \frac{\gamma_2\cos\varphi}{\varepsilon_2}\left[H_2^+ e^{-\gamma_2 d_1\cos\varphi} - H_2^- e^{\gamma_2 d_1\cos\varphi}\right]. \tag{11}$$

Dividing (11) by (10), and using (9) we can obtain the expression for the reflection coefficient at the boundary of the first and second dielectrics ($z = d_1$)

$$R_1 = \frac{H_1^-}{H_1^+} = \frac{1-q_1}{1+q_1} e^{-2\gamma_1 d_1\cos\psi} \tag{12}$$

where

$$q_2 = \frac{\varepsilon_1\gamma_2\cos\varphi}{\varepsilon_2\gamma_1\cos\psi}\left(\frac{1-R_2 e^{2\gamma_2 d_1\cos\varphi}}{1+R_2 e^{2\gamma_2 d_1\cos\varphi}}\right). \tag{13}$$

The similarity of (5), write the ratio between the values of the angles ψ and φ the expression (12) and (13)

$$\gamma_1\sin\psi = \gamma_2\sin\varphi\,. \tag{14}$$

At the boundary of air and the first dielectric layer, ($z = 0$) the boundary conditions for the electric field have the form of $E_0 = E_1$ when

$$\frac{\gamma_0\cos\theta}{\varepsilon_0}(H_0^+ - H_0^-) = \frac{\gamma_1\cos\psi}{\varepsilon_1}(H_1^+ - H_1^-)\,.$$

Dividing (14) by (6), taking into account (12), we can obtain the expression for the reflection coefficient $R_0 = H_0^- / H_0^+$ of the considered two-layer dielectric structure on a metal basis

$$R_0 = \frac{1-q_0}{1+q_0} \tag{15}$$

where

$$q_1 = \frac{e_0 g_1 \cos y}{e_1 g_0 \cos q}\left(\frac{1 - R_1}{1 + R_1}\right). \qquad (16)$$

From (6) and (14) we obtain

$$\sin y = \frac{g_0}{g_1}\sin q \; , \quad \sin j = \frac{g_0}{g_2}\sin q \, .$$

The latter relations allow us to express through a given angle q of incidence trigonometric functions $\cos y$ and $\cos j$, which are included in the expression for the reflection coefficients R_2, R_1, R_0

$$\cos y = \sqrt{1 - \left(\frac{g_0}{g_1}\right)^2 \sin^2 q} \; , \qquad (17)$$

$$\cos j = \sqrt{1 - \left(\frac{g_0}{g_2}\right)^2 \sin^2 q} \; . \qquad (18)$$

The common numerical solution of expressions (9), (12), (1.28), (15) – (18), allows to determine the reflection coefficient by the parameters of the structure and the angle of incidence of the electromagnetic wave, as well as to obtain the corresponding dependences.

In particular, in fig. 2 shows the dependence calculated for the modulus of the reflection coefficient R_0 on the angle of incidence ¯ for several values of the relative permittivity e_2ў of the second dielectric layer.

Model calculations were performed for the frequency of electromagnetic waves f = 8 GHz and the following values of the structure parameters: e_0ў = 1, tgd_0 = 0 (air); e_1ў = 2, tgd_1 = 0, d_1 = 5 mm (the first dielectric layer), e_2ў = 2 – 5, tgd_2 = 0,3, d_2 = 100 mm (the second dielectric layer).

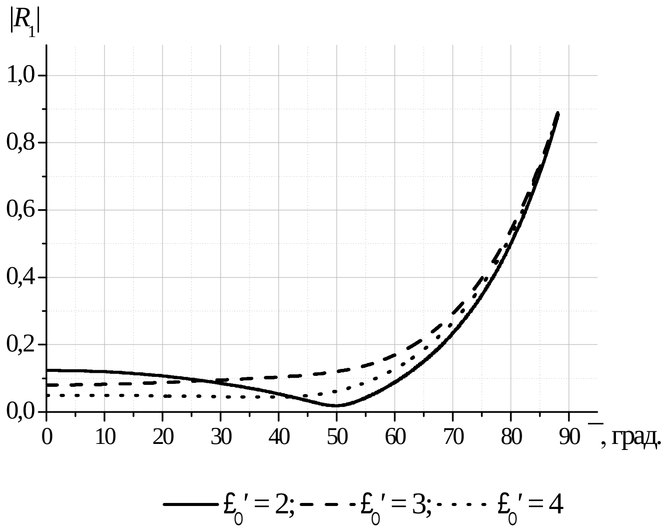

Fig. 2

As can be seen from fig.2, the largest range of change of the modulus of the reflection coefficient R_0 from the surface of the structure, when changing the relative dielectric constant of the second dielectric layer $e\check{y}_2$, is observed for the angles of incidence close to ¯ = 50 deg.

In particular, in the region of the angle of incidence ¯ = 50 deg. the range of change of the modulus of the reflection coefficient, with a change in the relative dielectric constant of the second dielectric layer, is much wider than with a normal incidence of the electromagnetic wave (¯ = 0 deg.).

More precisely, at a normal angle of incidence there is almost no sensitivity of the reflection coefficient R_0 to change $e\check{y}_2$.

Conclusions. A model of H-polarized wave propagation in a multilayer dielectric structure for an arbitrary angle of incidence was developed, and the limit of the range of change of the reflection coefficient at the change of the dielectric constant of its layers was determined.

The developed model makes it possible to determine the coefficient of reflection of electromagnetic waves from a multilayer dielectric structure by its parameters and the angle of incidence of the wave on the surface of the structure.

The model allows to obtain an analytical estimate of the range of the coefficient of change of reflection and reflection when changing the dielectric constant and thickness of each of the layers of the model of the dielectric structure on a metal basis.

1. *Borul'ko V. F., Drobakhin O. O., Slavin I. V.* Multifrequency Nondestructive Microwave Methods for the Characterization of Layered Dielectrics. Dnipropetrovsk: Dnipropetrovsk State University, 1992. 120 p. (*in Russian*).
2. *Shaarawi A. M., Besieris I. M., Attiya A. M., El-Diwan E.* Reflection and transmission of an electromagnetic X-wave incident on a planar air-dielectric interface: spectral analysis. Progress in electromagnetics research, PIER. 2001. V 30. P. 213–249. https://doi.org/10.2528/PIER00042502
3. *Kaz'min A. I., Fedyunin P. A.* Reconstruction of the structure of the electrophysical parameters of multilayer dielectric materials and coatings from the frequency dependence of the attenuation coefficient of the field of a surface electromagnetic wave. Measurement Techniques. 2019. V. 62. P. 809–816. https://doi.org/10.1007/s11018-019-01699-7
4. *Lytvynenko L. M., Prosvirnin S. L.* Wave Diffractionby Periodic Multilayer Structures. – Cottenham. UK: Cambridge Scientific Publishers, 2012. 158 p.
5. *Gerekos C., Tamponi A., Carrer L., Castelletti D., Santoni M., Bruzzone L. A* Coherent Multilayer Simulator of Radargrams Acquired by Radar Sounder Instruments. IEEE Trans. on Geoscience and Remote Sensing. 2018, Vol. 56, Iss. 12. Pp. 7388–7404. https://doi.org/10.1109/TGRS.2018.2851020
6. *Kaliberda M. E., Lytvynenko L. M., Pogarsky S. A.* The h-polarized electromagnetic wave diffraction by multi-element plane semi-infinite grating. Radio physics and radio astronomy. 2014. Vol. 19, № 4. Pp. 348–357 (in Russian). https://doi.org/10.15407/rpra19.04.348
7. *Chen L. F, Ong C. K. , Neo C. P. , Varadan V. V., Varadan V. K.* Microwave Electronics: Measurement and Materials Characterization. Pennsylvania State University, USA, New York: John Wiley & Sons, Ltd, 2004. 537 p. https://doi.org/10.1002/0470020466
8. *Lebedev, I. V.* Microwave equipment and devices. Moscow: Higher School, 1972. 374 p. (in Russian).

Received 25.02.2021,
in final form 17.03.2021