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D. N. LAZUCHENKOV, N. M. LAZUCHENKOV

CALCULATION OF THE ION CURRENT TO A CONDUCTING CYLINDER IN A SUPERSONIC FLOW OF A COLLISIONLESS PLASMA.

Th Institute of Technical Mechanics of
National Academy of Sciences of Ukraine and State Space Agency
of Ukraine, 15 Leshko-Popelya St., Dnipro, 49600, Ukraine; e-mail: lazuch.dn@gmail.com

В основі діагностики потоків низькотемпературної плазми циліндричними зондами лежить класичне співвідношення Ленгмюра для іонного струму на тонкий відносно дебаївського радіуса циліндр. Метою статті є дослідження застосовності співвідношення Ленгмюра для циліндра з радіусом, перевищуючим дебаївський радіус екранування.

Виконано числове моделювання взаємодії провідного циліндра з потоком розрідженої плазми. Циліндр знаходиться під негативним потенціалом щодо плазми. Моделювання вільномолекулярного обтікання проведено на основі двовимірної системи рівнянь Власова–Пуассона. При розрахунку відштовхуючого електрони локально рівноважного самоузгодженого електричного поля використана модель Пуассона–Больцмана в наближенні локально рівноважних електронів і з урахуванням стоку електронів на поверхні циліндра в наближенні центрального поля. Рівняння Власова для іонів і Пуассона–Больцмана для самоузгодженого електричного поля розв’язані на вкладених сітках кінцеворізницевим методом установлення з розщепленням за фізичними процесами і використанням методу характеристик. Вірогідність отриманих результатів підтверджено розв’язанням відомих модельних задач, порівнянням з результатами розрахунків інших авторів і результатами розв’язання однакових фізичних задач з використанням різних математичних моделей і методів.

Розраховано іонні струми на поперечно обтічний циліндр у залежності від його потенціалу, іонного швидкісного відношення і характерного розміру циліндра відносно дебаївського радіусу екранування. За результатами числового моделювання отримані кількісні оцінки області застосовності класичного співвідношення Ленгмюра для іонного струму на циліндр із радіусом, перевищуючим дебаївський радіус екранування. Отримані результати можуть бути використані в діагностиці надзвукових потоків низькотемпературної розрідженої плазми.

***Ключові слова*:** *потік розрідженої неізотермічної плазми, поперечне вільномолекулярне обтікання довгого кругового циліндру, числове моделювання, система рівнянь Власова–Пуассона, розрахунок іонного струму на циліндр.*

The diagnostics of low-temperature plasma flows using cylindrical probes is based on the classical Langmuir relation for the ion current to a thin, in comparison with the Debye length, cylinder. The aim of this work is to study the applicability of the Langmuir relation for a cylinder whose radius exceeds the Debye length.

The interaction of a conducting cylinder with a rarefied plasma flow was simulated numerically. The cylinder had a negative potential with respect to the plasma. Free molecular flow around the cylinder was simulated on the basis of a two-dimensional system of the Vlasov–Poisson equations. The electron-repulsing local equilibrium self-consistent electric field was calculated using the Poisson–Boltzmann model in the approximation of local equilibrium electrons and taking into account an electron sink on the cylinder surface in the central field approximation. The Vlasov equations for ions and the Poisson–Boltzmann equations for the self-consistent electric field were solved on nested grids by a finite-difference relaxation method with splitting by physical processes and using the method of characteristics. The reliability of the calculated results was confirmed by the solution of known model problems and a comparison with the results of other authors and the results of solving identical physical problems with the use of different mathematical models and methods.

The ion current to a cylinder placed transversely to a plasma flow was calculated as a function of the cylinder potential, the ion velocity ratio, and the ratio of the characteristic dimension of the cylinder to the Debye length. From the calculated results, numerical estimates were obtained for the range of applicability of the classical Langmuir relation for the ion current to a cylinder whose radius exceeds the Debye length. The results obtained may be used in the diagnostics of supersonic flows of a low-temperature rarefied plasma.

Keywords: rarefied nonisothermal plasma flow, transverse free molecular flow around a long circular cylinder, numerical simulation, system of Vlasov–Poisson equations, calculation of the ion current to a cylinder.

Introduction. The probe method of plasma diagnostics is still in demand as a simple and reliable method for determining the parameters of low-temperature plasma. Due to the simplicity of the equipment and acceptable measurement accuracy, stationary cylindrical Langmuir probes are successfully used to study the kinetic parameters of the charged components of the ionospheric plasma, at laboratory modeling of ionospheric conditions, testing and calibrating scientific on-board equipment at the developing stage in space experiments, and also to control technological plasma processes.

The main elements of a probe measuring system are a measuring electrode (probe) with a current-collecting surface area S_p and a reference electrode having electrical contact with the plasma on the surface with an area of S_{cp}. For the measurement scheme of a single Langmuir probe, the ratio of the reference electrode to probe areas $S_s = S_{cp}/S_p$ must satisfy a rather strict condition $S_s \geq 10^4$, which is not always easy to meet when using the spacecraft body as a reference electrode. In such a situation, it is proposed to use an isolated probe system that is not electrically connected to the spacecraft body [1].

The ion current collected by the cylindrical electrode is three orders of magnitude smaller than the electron current. Therefore, when diagnosing a rarefied plasma using the ionic part of the current-voltage characteristic, the area of the current-collecting surface and the negative potential of the probe relative to the plasma must be large enough for reliable measurement of ion current.

When interpreting measurements with cylindrical probes, the asymptotic relation obtained by I. Langmuir and H. Mott-Smith [2] which was developed further in [3] for the ion current collected by a thin cylinder in a collisionless plasma flow is used:

$$\bar{I}_i(\varphi_c) = 2/\sqrt{\pi}\sqrt{1/2 + S_i^2 - \beta\varphi_c}, \quad \varphi_c < S_i^{\,2}/\beta, \tag{1}$$

where $\bar{I}_i$ is the ion current on the cylinder related to the thermal current of ions, $\beta = T_e/T_i$ is the ratio of the temperatures of electrons T_e and ions T_i, $S_i = V/u_i$ is the ion velocity ratio, V is the mass velocity of the plasma flow, $u_i = \sqrt{2kT_i/m_i}$ is the thermal velocity of the ions, m_i is the ion mass, φ_c is the dimensionless potential of the cylinder relative to the unperturbed plasma potential (electric potential is normalized by kT_e/e, where k stands for the Boltzmann constant, e is the elementary charge). Here and below, the index i corresponds to ions, e – to electrons. Relation (1) is obtained under the assumption that the ratio of the characteristic dimensions of the body and plasma $\xi = r_c/\lambda_D$ (r_c is the cylindrical electrode base radius, λ_D is the Debye length) is small, i.e. $\xi \ll 1$.

In [4] a procedure was developed for determination the flow parameters of a collisionless plasma with two ion sorts by the current-voltage characteristic of an isolated probe system with cylindrical electrodes for an arbitrary probe-to-reference electrode surface areas ratio S_s. It is shown in [5] that, in order to reliably determine the electron density and temperature together with the ionic composition of the plasma with two-sorted ions, measurements are required at probe bias potential up to several tens of volts relative to the reference electrode potential. Fig. 1 represents in a dimensionless form the dependence of the equilibrium potential of the reference electrode φ_{cp} on the probe bias potential φ_{iz} relative to the reference electrode for various values S_s of the electrodes area ratio in the probe system. It can be seen that at $S_s < 200$ large values of the bias potential φ_{iz} lead to large negative potentials of the reference electrode with respect to the plasma.

The base radius of the cylindrical reference electrode of the probe system r_{cp} must be significantly larger than the base radius of the probe r_p. For the optimal choice of the characteristic dimensions of cylindrical electrodes, it is necessary to know the applicability limits of relation (1) for the velocity ratio S_i, cylinder potential φ_c and parameter ξ.

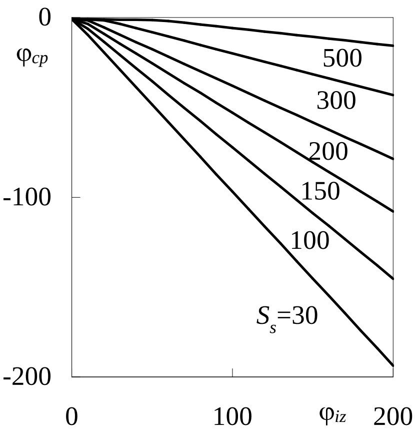


Fig. 1

It is known that to model the interaction of a charged body with a rarefied plasma at nonrelativistic velocities in the absence of an external magnetic field, it is sufficient to use the self-consistent system of Vlasov–Poisson equations. Results of numerical calculations of ion current collection in a free molecular flow around a cylinder, obtained in the approximation of a symmetric electric potential field, are presented in [6]. The flow of rarefied plasma around a cylinder that is “large” relative to the Debye length, usually involves an asymmetric spatial distribution of the potential around it, which complicates the calculation of the parameters of unperturbed plasma. Ion current on the cylinder in asymmetric potential field is calculated in [7] on the basis of the two-dimensional stationary Vlasov–Poisson model with Boltzmann electron distribution. Calculations of ion current on the cylinder using the two-dimensional Vlasov–Poisson model, which includes the kinetic equation for electrons, is performed in [8]. Based on the results of these calculations, relation (1) for ion current on the cylinder in a collisionless nonmagnetic plasma flow is estimated to be applicable for negative cylinder potential φ_c of up to -25, for parameter ξ from 1 to 100, and a velocity ratio S_i from 0 to 10.

Results of numerical calculations of ion current collection in a free-molecular transverse flow around a cylinder, obtained using a two-dimensional Vlasov–Poisson model using model electron distributions at negative cylinder potential of up to $\varphi_c = -60$, parameter ξ from 1 to 3, and velocity ratio S_i from 1 to 7 are presented in [9]. The current article represents the results of numerical calculations of the ion current on the cylinder at negative potentials of up to $\varphi_c = -200$, parameter ξ from 1 to 10, and velocity ratio S_i from 3 to 7.

Formulation of the problem. The interaction of an infinitely long conducting circular cylinder with a transverse flow of three-component rarefied nonisothermal plasma consisting of neutrals, positive singly charged ions and electrons is modeled. Let the plasma be Maxwellian, quasi-neutral, the flow around the cylinder is free-molecular, the influence of the magnetic field is negligible. Flow velocity V, ion density n_0 in the undisturbed flow, and the potential of the cylinder φ_c relative to the potential of the undisturbed plasma are known. The mean free paths of all plasma components significantly exceed the characteristic transverse dimensions of the cylinder, and the conditions $Kn >> Ma >> 1$ are satisfied for Knudsen and Mach numbers. It’s assumed that the surface of the cylinder completely absorbs the charge of contacting particles (electrons are absorbed, ions are neutralized), and there is no electron emission.

Let's introduce a rectangular Cartesian coordinate system in the physical space $Oxyz$, whose axis Oz is directed along the axis of the cylinder, the axis Ox – in the direction of the flow velocity. In this case, the velocity distribution functions of charged particles are five-dimensional, determined by a point in the four-dimensional phase space and time. The surface of a cylinder of unit length is modeled by a circle centered at the origin. The potential of the cylinder relative to the potential of the unperturbed plasma is negative ($\varphi_c < 0$).

Under the mentioned assumptions, the space-time evolution of the distribution functions of charged plasma particles is primarily determined by the self-consistent electric field and it is described by the Vlasov–Poisson mathematical model. In the accepted coordinate system in dimensionless quantities, the system of Vlasov–Poisson equations writes [9, 10]:

$$\frac{\partial f_i}{\partial t} + \mathrm{v}\frac{\partial f_i}{\partial \mathrm{x}} - \beta\frac{1}{2}\frac{\partial \varphi}{\partial \mathrm{x}}\frac{\partial f_i}{\partial \mathrm{v}} = 0\,, \tag{2}$$

$$\Delta\varphi = -\xi^2\left[n_i - n_e(\mathrm{x},\varphi)\right], \quad n_i = \int\limits_{\overline{\Omega}_\mathrm{V}} f_i d\mathrm{v}\,. \tag{3}$$

Here $\mathrm{x} = (x, y)$ are coordinates in the physical space, normalized by the base radius of the cylinder r_c; $\mathrm{v} = (v_x, v_y)$ are the coordinates in the space of ion velocities, normalized by the thermal velocity u_i; t – time, normalized by r_c/u_i; β is the ratio of temperatures of charged plasma particles; ξ is the ratio of the cylinder radius to the Debye length; φ is the potential of the electric field, normalized by kT_e/e; $f_i(t,\mathbf{x},\mathbf{v})$ is the ion velocity distribution function, normalized by n_0/u_i^2; n_i is the density of ions; $\overline{\Omega}_\mathrm{V}$ is the computational domain in the space of dimensionless ion velocities; $n_e(\mathrm{x},\varphi)$ is the model distribution of the electron density. The densitys of ions and electrons are normalized by the density in the unperturbed plasma n_0.

The ion velocity distribution function f_i and potential φ satisfy the following boundary conditions:

$$f_i\big|_{t=0} = f_i^0; \quad f_i\big|_{|\mathrm{x}|\to\infty} = f_i^\infty; \quad f_i(t,\mathrm{x},\mathrm{v})\Big|_{\substack{\mathrm{x}\in\partial\overline{\Omega}_0 \\ (\mathrm{v},\mathrm{n})>0}} = 0\,, \tag{4}$$

$$\varphi\big|_{|\mathrm{x}|\to\infty} = 0\,, \quad \varphi\big|_{\partial\overline{\Omega}_0} = \varphi_c\,. \tag{5}$$

Here, n is the outer normal to the surface of the cylinder $\partial\overline{\Omega}_0$. On the symmetry surface of the problem $y = 0$, the conditions for specular reflection are specified for the ion distribution function, and the conditions for the symmetry of the electric field for the potential. Far from the body, the velocity distribution of ions is Maxwellian. The initial ion distribution function $f_i^0(\mathrm{x},\mathrm{v})$ is specified using the analytical solution of the problem of a neutral rarefied gas flow around a cylinder [10] or the previously obtained solution of the problem under similar flow conditions.

In an unperturbed plasma far from the body, electrons that are much lighter than other components are locally in equilibrium, and their density is described by the Boltzmann distribution $n_e(\mathrm{x},\varphi)=e^{\varphi}$. In the vicinity of cylinder in a retarding field, approximate solutions of the Vlasov kinetic equations for Maxwellian electrons in a central field [11] were used as model electron distributions $n_e(\mathrm{x},\varphi)$.

Problem solving method. The Vlasov kinetic equation (2) and the Poisson–Boltzmann equation (3) are solved by the finite difference method in rectangular coordinate systems in the physical space and velocity space [9]. In the physical space, uniform nested grids are used, in the space of velocities, a uniform grid with cell size providing the necessary accuracy of approximation of the distribution function is used. Non-uniform grids are used near the surface of the cylinder in physical space and near the discontinuity surfaces in velocity space. The position of the discontinuity surface of the distribution function in the velocity space was corrected by the method of characteristics.

The computational domains in the physical space and in the velocity space are chosen large enough to cover the region of the perturbation of the plasma flow and the electric field by the charged cylinder. At the boundaries of the spatial computational domain, the corresponding physical boundary conditions (4) – (5) are set. At high flow velocities in the areas where the wake goes beyond the computational domain boundary, an artificial boundary conditions are used [9].

The general scheme for solving problem (2) – (3) is reduced to two iterative processes: 1) solution of the Vlasov kinetic equation for ions (2) for a given electric potential field; 2) solution of the equation for the self-consistent electric field (3) for a given ion density field. The second iterative process is nested inside the first one. In each iterative process, to smooth nonphysical changes in the parameters of the problem, the simplest regularizing operators are used, which are based on the physical features of the problem like the non-negativity of the distribution function, minimum energy of the self-consistent electric field, and the regions of monotonicity of the electric field.

The Vlasov equation (2) is solved by the method of setting with splitting by physical processes and using the method of characteristics, the Poisson–Boltzmann equation (3) for a self-consistent electric field is solved by the iterative method. It is known that local perturbations of the distribution function have little effect on macroparameters. Therefore, the recalculation of the equilibrium self-consistent electric field is carried out once per several iterations of the solution of the Vlasov equation or after a noticeable integral change in the ion density at characteristic locations such as the space charge region and the wake behind the cylinder. Iterations in the calculation of the potential on each of the nested grids are interrupted when the specified calculation accuracy is reached, or when the linear rate of convergence is reached.

The consistency of the obtained electric field and ion velocity distribution function is controlled by calculating the distribution function momentum at a number of characteristic points near the cylinder surface using method of characteristics. Once the system of Vlasov–Poisson equations (2) – (3) is solved, the calculation of the ion current density per unit length is carried out by integrating the first momentum of the ion distribution function over the cylinder contour.

Calculation results. The reliability of the results of solving the Vlasov–Poisson system (2) – (3) is confirmed by carrying out test calculations of a number of model problems, the solutions of which are known and verified by numerous experiments, comparison with the results of calculations by other authors and the results of solving identical physical problems using different mathematical models and methods [9]. The result of calculation of the collected ion current depending on the potential of the cylinder φ_c at $\xi = 10$, $\beta = 1$ for various S_i, along with the results from [6 – 8], is shown on Fig. 2. The ion current $\bar{I}_i$ is normalized by the thermal ion current. The curves in the figure correspond to the calculations: 1 – authors, 2 – Godard, Laframboise [6], 3 – Xu [7], 4 – Choiniere [8]. Calculated ion current fits satisfactorily to the results of solving two-dimensional Vlasov–Poisson system using the approximation of Boltzmann electrons [7] and using the kinetic equation for electrons [8]: for all considered flow velocities, the discrepancy in ion current is no more than 3 %. At the velocity ratio of $S_i \geq 5$, the obtained value of ion current on the cylinder also coincided with the results of calculations in [6] with satisfactory accuracy. At $S_i = 3$, the ion current calculated in [6] using the approximation of a symmetric electric field differs significantly from the presented here result obtained using the Vlasov–Poisson model. Thus, the two-dimensional mathematical model of Vlasov–Poisson (2) – (3) makes it possible to carry out numerical simulation of current collection by a long cylinder in a supersonic rarefied plasma flow with acceptable accuracy for diagnostics.

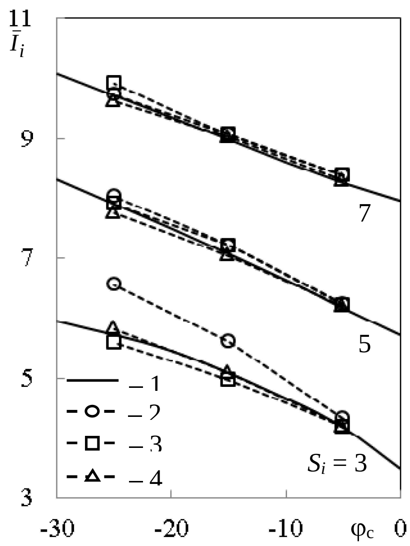


Fig. 2

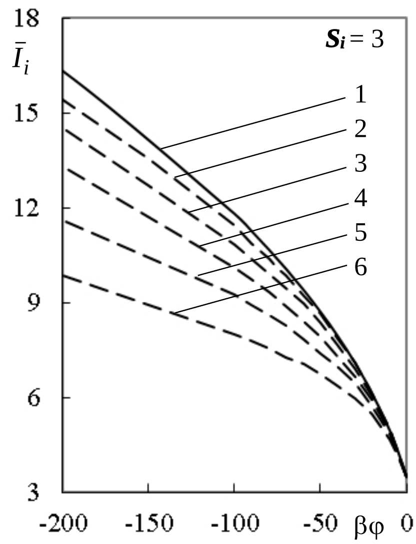


Fig. 3

The influence of parameters ξ, S_i, β on the ion current collected by the conducting cylinder placed transversely in the supersonic collisionless plasma flow is carried out. Dimensionless potential of the cylinder relative to plasma φ_c varied from 0 to -200. Results of calculation of the dimensionless ion current depending on the parameter $\beta\varphi$ for $\xi = 1, 3, 5, 7, 10$ are presented on Fig. 3 at $S_i = 3$, Fig. 4 at $S_i = 5$ and Fig. 5 at $S_i = 7$. Curves on Fig. 3 – 5 stand for: 1 – asymptotic relation (1); 2 – calculation at $\xi = 1$; 3 – at $\xi = 3$; 4 – at $\xi = 5$; 5 – at $\xi = 7$; 6 – at $\xi = 10$.

Calculations of collected ion current at $\beta = 1$, 1.5, 2 revealed weak dependency of the current on β. At low negative potential of the cylinder relative to the plasma potential, relation (1) approximates well the collected ion current for all considered range of ξ and S_i. The calculated ion current qualitatively correspond to the known dependencies: an increase in the parameter ξ leads to a decrease in the ion current; as the velocity ratio S_i increases, the influence of the parameter ξ on the dimensionless ion current wanes.

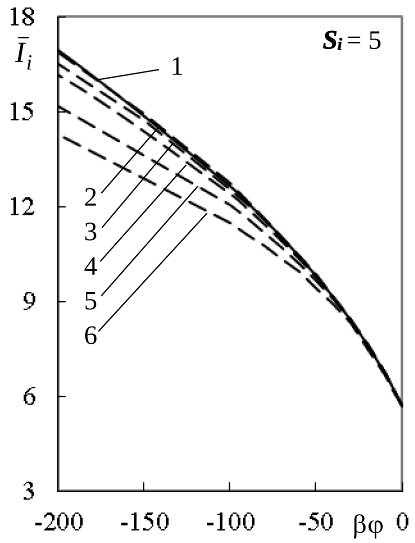


Fig. 4

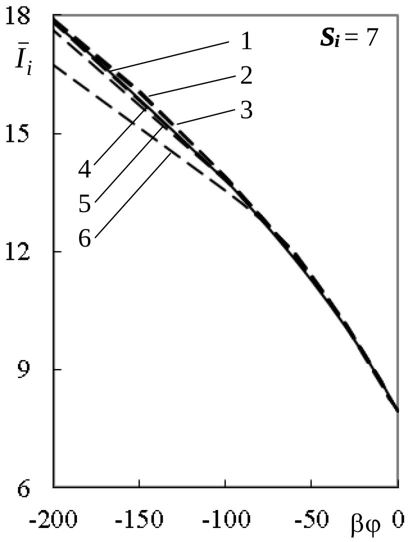
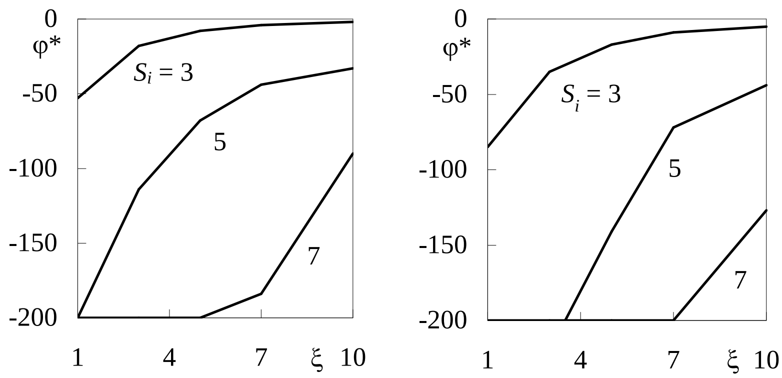


Fig. 5

The analysis of the obtained results of numerical calculations made it possible, within the framework of the accepted assumptions, to estimate for various ξ and S_i such range of potential $\varphi^*(\xi, S_i) \le \varphi_c \le 0$ for which relation (1) is applicable.

The lowest limit potential φ^* (for given ξ and S_i), for which the discrepancy between the ion currents calculated by (1) and by Vlasov–Poisson model doesn't exceed 1 % is presented on Fig. 6 (a), and Fig 6 (b) represents that for 3 % difference in ion currents.



a) b)

Fig. 6

Conclusions. Numerical modeling of interaction between the supersonic rarefied nonequilibrium plasma flow and conducting cylinder under a negative potential with respect to plasma is carried out on the basis of the two-dimensional system of Vlasov– Poisson equations. The Vlasov equations for ion parameters and the Poisson–Boltzmann equations for a self-consistent electric field are solved on nested grids by the finite difference method of setting with splitting into physical processes.

The dependences of the collected ion current on the plasma flow velocity, potential and characteristic cylinder size relative to the Debye length are obtained. By the results of numerical simulation, quantitative estimates are obtained for the applicability of the classical Langmuir relation for the ion current on a cylinder in the case of base radius equal to or greater than the Debye length.

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