

WAYS TO CONSTRUCT THE MAXIMUM INTERPOINT DISTANCE DISTRIBUTION FOR RANDOM NORMAL POINTS IN A PLANE

*M. K. Yangel Pivdenne State Design Office
3 Kryvorizka St., Dnipro 49008, Ukraine; -mail: edgladky@gmail.com*

Many practical problems call for constructing the maximum interpoint distance distribution for random points in a plane. In the literature, the case of a great number of points is considered, for which an asymptotic distribution is determined. This paper addresses the problem of constructing the maximum interpoint distance distribution for a small number of random points in a plane whose coordinates are independent random quantities that obey the standard normal distribution. The special case of three random points in a plane is considered as the basic one, for which three ways to construct the maximum interpoint distance distribution are studied.

The first way is to construct the distribution function from geometrical considerations. To do this, the loci of three points are considered from the condition that the maximum distance between them shall not exceed a certain value. The position of the third point in the plane is determined relative to the two other points: the leftmost and the lowermost one. In this case, the construction of the distribution function involves the successive evaluation of several integrals using numerical methods. The obtained results are in good agreement with those of statistical simulation.

The second way is based on studying the distance between pairs of random normal points in a plane. Taken separately, the distances between each pair of random normal points obey one-dimensional Rayleigh distributions, but in the aggregate they prove to be correlated because they are determined from the same point coordinates. A joint distribution of the squared distances between three points is constructed using the three-dimensional Moran-Downton distribution. Using it, a distribution function of the squared maximum distance between three random normal points, which is identical with the maximum interpoint distance distribution, is obtained. It is found that for small values it underestimates the actual probability of the maximum distance not exceeding a certain value. For great distance values, the above probabilities coincide.

The third way uses the Rice distribution (a generalization of the Rayleigh distribution) to approximate the unknown maximum interpoint distance distribution for three random normal points in a plane. The Rice distribution parameters found by the least-squares method are in good agreement with those obtained by statistical simulation.

The results for three random normal points are generalized to a greater number of points (up to 30). It is shown that in this case the third way is most efficient.

Keywords: *random points in a plane, maximum interpoint distance, distribution function, Moran–Downton distribution, Rice distribution.*

1. Gubarev V. V. Tables of Random Variable and Vector Characteristics. Novosibirsk: Novosibirsk Institute of Electrical Engineering, 1980. 225 pp. (in Russian).
2. Gumbel E. Statistics of Extremes. Moscow: Mir, 1965. 452 pp. (in Russian).
3. David G. Order Statistics. Moscow: Nauka, 1979. 336 pp. (in Russian).
4. Kramer G. Mathematical Methods of Statistics. Moscow: GIL, 1975. 648 pp. (in Russian).
5. Appel M. J. B., Najim C. A., Russo R. P. Limit laws for the diameter of a random point set. 2002. Adv. Appl. Probab. V.34. Pp. 1-10
<https://doi.org/10.1017/S0001867800011356>
6. Joarder A. H., Omar M., Gupta A. K. The distribution of a linear combination of two correlated chi-square variables. Revista Colombiana de Estadística. 2013. Vol. 36. No. 2. Pp. 211-221.
7. Jonson N. L., Kotz S., Balakrishnan N. Continuous Univariate Distributions. Volume 1. New York: John Wiley and Sons, 2000. 756 pp.
8. Kotz S., Balakrishnan N., Johnson N. L. Continuous Multivariate Distributions. Volume 1: Models and Applications. New York: John Wiley and Sons, 2000. 722 pp.
<https://doi.org/10.1002/0471722065>

9. Matthews P. C., Rukhin A. L. Asymptotic distribution of the normal sample range. 1993. Ann. Appl. Probab.
V. 3. No. 2. Pp. 454-466.
<https://doi.org/10.1214/aop/1177005433>

Received on April 5, 2024,
in final form on June 2, 2024