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S. Ye. MALYSHEV[1], K. V. AVRAMOV[2]

NONLINEAR MODES OF A NONLINEARLY DEFORMED BEAM WITH A BREATHING CRACK

[1] *National Technical University «Kharkiv Polytechnical Institute»*
2 Kyrpychova St., Kharkiv 61002, Ukraine
[2] *A. Pidgorny Institute of Mechanical Engineering Problems*
of the National Academy of Sciences of Ukraine
2/10 Komunalnykiv St., Kharkiv 61046, Ukraine; e-mail: kvavr@kharkov.ua

Two types of partial differential equations, which describe geometrically nonlinear vibrations of a beam with a breathing crack, are derived. Thus, two sources of nonlinearities are considered. The crack function is used to describe the 3D strain state near the crack in the first model. Delta functions are used to describe the crack in the second model. The Hu-Washizu variational principle is used to derive the partial differential equations of the first model. The Hamilton principle is used to derive the partial differential equations for the second model. The obtained partial differential equations are reduced to integro-differential ones by neglecting the longitudinal inertia and accounting for the boundary conditions. A contact parameter is used to describe the nonlinear breathing of the crack. The Galerkin technique is used to obtain a nonlinear system of ordinary differential equations with both polynomial nonlinearity and piecewise linear functions. To study nonlinear vibrations numerically, the collocation method is used together with an algorithm of solution continuation along the arclength using an automatic differentiation technique, which allows one to combine the accuracy of analytical differentiation with the simplicity of numerical differentiation. is used to analyze numerically nonlinear oscillations. A monodromy matrix and its eigenvalues, which are called multipliers, are calculated to analyze the stability and bifurcations of the periodic motions. The backbone curves of nonlinear modes contain two loops, saddle-node bifurcations, and Naimark-Sacker bifurcations. As follows from the numerical analysis, the nonlinear modes in the configurational subspace are essentially curved. Moreover, the nonlinear modes on the backbone curve loops have an oscillating appearance in configurational subspace. These loops may be indicative of closed loops of forced vibrations.

Keywords: nonlinear vibrations of curved beams, breathing crack, Galerkin technique, nonlinear modes, Naimark-Sacker bifurcation.

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