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THE EFFECT OF SOLAR RADIATION PRESSURE ON THE MOTION OF SATELLITES IN ALMOST CIRCULAR EARTH ORBITS

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Розглянуто вплив тиску сонячної радіації на рух супутника майже круговими низькими навколоземними орбітами. Постановка задачі зумовлена необхідністю визначення впливу сил тиску сонячної радіації на рух легких комерційних супутників дистанційного зондування Землі (ДЗЗ) з великими площами їх поверхонь (сонячні батареї, антени). Задача полягає у визначенні основних закономірностей цього впливу, побудові досить простих і достатньо точних оцінок зміни параметрів орбіт для орбіт, що розглядаються, і з'ясуванні фізики (причинно-наслідкових зв'язків) змін параметрів. Новизна дослідження також полягає у використанні змінних, що спеціально введено для опису руху майже круговими орбітами.

Дослідження проводяться в припущенні постійності сили тиску сонячного випромінювання по всій орбіті і безпосередньо пов'язані з орбітами «світанок–захід», які часто використовуються для супутників ДЗЗ із системами радіолокаційного спостереження.

У результаті дослідження були побудовані прості аналітичні вирази, що описують основні закономірності короткочасних (декількоденних) змін параметрів орбіти. Показано, що зміна орієнтації площини орбіти визначається дією гіроскопічного моменту. Цей момент врівноважує дію моменту зовнішніх сил, спрямованих на зміну орієнтації, і зміну орієнтації перпендикулярно напрямку прикладеного моменту зовнішніх сил. Основним ефектом дії сил тиску сонячної радіації є збудження вимушених коливань радіуса орбіти з амплітудою, що лінійно зростає з часом. Максимуми цих коливань (апогей) припадають на точку, де сили світлового тиску максимально сповільнюють рух супутника (спрямовані проти швидкості), а мінімуми (перигей) – у точці максимального прискорення руху.

Показано, що річний рух Сонця може якісно змінити картину еволюції параметрів орбіти. Для сонячно-синхронних орбіт "світанок–захід" побудовано компактні аналітичні розв'язки для зміни параметрів орбіти та показано, що річний рух схилення Сонця призводить до зміни напрямку еволюції форми орбіти на протилежний.

Проведені розрахунки показали достатньо високу точність аналітичних рішень на початковому інтервалі часу. Отримані чисельні оцінки дають змогу оцінити вплив сонячного тиску на зміну параметрів орбіти.

Ключові слова: *тиск сонячної радіації, супутник дистанційного зондування Землі, основні закономірності, аналітичні рішення, зміна форми орбіти*.

This paper considers the effect of the solar radiation pressure on the motion of a satellite in an almostcircular low-Earth orbit. The formulation of the problem is due to the need to determine the effect of solar radiation pressure forces on the motion of light commercial Earth remote sensing (ERS) satellites with large surface areas (solar batteries and antennas). The goal is to determine the main regularities of this effect, construct reasonably simple and accurate estimates of changes in orbital parameters for the orbits under consideration, and clarify their physics (cause-and-effect relations) The novelty of this study also lies in the use of variables specially introduced to describe a motion in almost circular orbits.

The study assumes that the solar radiation pressure force is constant throughout the entire orbit, and it is concerned with dawn-dusk orbits, which are often used for ERS satellites with radar observation systems.

The paper presents simple analytical expressions that describe the main regularities of short-term (several days) changes in orbital parameters. It is shown that the change in the orientation of the orbital plane is determined by the action of the gyroscopic moment. This moment balances the effect of the moment of external forces aimed at changing the orientation and the change in the orientation perpendicular to the direction of the applied moment of the external forces. The main effect of the solar radiation pressure is the excitation of forced oscillations of the orbital radius, whose amplitude linearly increases with time. The maximums of these oscillations (apogee) are at the point where the light pressure forces maximally slow down the motion of the satellite (directed oppositely to the velocity), and the minimums (perigee) are at the point of the maximum motion acceleration.

It is shown that the annual movement of the Sun can qualitatively change the picture of the evolution of orbital parameters. For sun-synchronous dawn-dusk orbits, compact analytical solutions for changes in orbital parameters are constructed, and it is shown that the annual movement of the Sun's declination reverses the direction of evolution of the orbital shape.

The calculations showed a reasonably high accuracy of the analytical solutions at the initial stage. The obtained numerical estimates make it possible to evaluate the effect of the solar pressure on changes in orbital parameters.

Keywords: *solar radiation pressure, Earth remote sensing satellite, main regularities, analytical solutions, change in orbital shape.*

Introduction. The effect of solar radiation pressure on the motion of a satellite in almost circular low Earth orbits is considered. Interest in this issue is associated with the need to assess the effect of various disturbances on the movement of Earth remote sensing (ERS) satellites. As is known, remote sensing satellites use low Earth orbits, for which the main disturbing factor is the non-centrality of the Earth's gravitational field. However, the desire to increase energy availability and the corresponding increase in the area of solar panels, as well as the desire to reduce the mass of satellites, are increasing interest in studying the effect of solar radiation pressure on the movement of ERS satellites. We also note the increasing requirements for the accuracy of the movement of ERS satellites, and the task of choosing stable (frozen) orbits [1]. Therefore, studies of the effect of solar radiation pressure on the motion of satellites in almost circular orbits are relevant.

Extensive studies of the effect of solar radiation pressure on the motion of artificial Earth satellites are associated with significant changes in the orbital parameters (orbital eccentricity) of the Vanguard I satellite (1958) [2]. In the article [3], using Lagrange's planetary equations, the effects of solar radiation pressure on the orbital parameters of the Earth's satellites were studied, and equations for changes in parameters averaged over the true anomaly were obtained. Since then and to the present, numerous studies have been carried out on various issues of the effect of solar radiation pressure on the movement of the Earth's satellites (see, for example, [4–8]). However, formulas describing the effect of solar radiation pressure on changes in orbital parameters in the general case are very cumbersome [9, 10]. Therefore, it is of interest to obtain simpler formulas for changing the parameters of almost circular orbits. For ERS satellites, studies of sun-synchronous orbits (SSO) are of particular interest, because in this case, the change in the position of the satellite's orbital plane is commensurate with the movement of the average Sun (the case of possible resonance [3, 9]). Also, for control and measurement problems, short-period oscillations of orbital parameters are of interest, as well as estimates of changes in orbital parameters over several days, when the position of the Sun can be considered constant.

In the article, under the assumption that the force of solar radiation pressure acting on the satellite is constant along the orbit, the main regularities of changes in the parameters of almost circular orbits are determined, and analytical expressions are constructed that quite accurately describe the change in the orbital parameters of the satellite over a short time interval (up to several days). Simple estimates of changes in parameters for sun-synchronous "sunrise-sunset" orbits are constructed, and the effect of the Sun's movement on changes in parameters is shown. The novelty of the research also lies in the use of variables specially introduced to describe motion in almost circular orbits [11, 12].

Formulation of the problem. The formulation of the problem is due to the need to determine the effect of solar radiation pressure forces on the movement of light commercial ERS satellites with large areas of their surfaces (solar batteries, antennas). It is required to determine the main regularities of this effect, to construct fairly simple and sufficiently accurate for the orbits under consideration estimates of changes in orbital parameters, and to clarify the physics (cause-and-effect relations) of changes in parameters. Particular attention is paid to sun-synchronous orbits, which are often used by ERS satellites.

Nearly circular orbits are understood as orbits whose radius changes are fractions of a percent of the orbital radius. It is these orbits that are used in most cases in ERS missions.

Research is carried out under the assumption of constant solar radiation pressure forces throughout the entire orbit. It seems that taking into account the Earth's shadow on the satellite's motion would lead the research quite far from its goal of obtaining simple estimates. You can verify this by looking at the history of the description of the shadow function (see, for example, [13–15]). Taking into account the Earth's shadow, as well as the effect of changes in orientation and area of effect of solar radiation pressure on the satellite as it moves in orbit, require additional research. The research carried out in the article directly relates to the "dawn-sunset" orbits, which are often used for ERS satellites with radar observation systems.

Coordinate systems. To describe the motion of the satellite we use the following rectangular coordinate systems:

– inertial coordinate system (ICS) $O_E XYZ$ with the origin at the Earth's center of mass (point O_E), the axis $O_E X$ is directed to the vernal equinox point, the axis $O_E Z$ is along the Earth's rotation axis, the axis $O_E Y$ complements the system to the right. We denote the unit vectors of this system as $\vec{e}_X$, $\vec{e}_Y$, $\vec{e}_Z$ respectively;

– orbital coordinate system (OCS) with the origin at the satellite's center of mass (point O), the Ox axis is directed along the radius vector of the satellite's center of mass; Oy axis is in the plane of the instantaneous orbit in the direction of the satellite's movement; The Oz axis is binormal to the orbit. The unit vectors of this system, directed along the radius vector of the satellite, transversals and binormals to the orbital plane will be denoted by $\vec{e}_r$, $\vec{e}_\tau$, $\vec{e}_n$ respectively;

– geocentric coordinate system $O_E X_S Y_S Z_S$ associated with the direction to the Sun: the axis $O_E X_S$ is directed to the Sun, the axis $O_E Z_S$ is perpendicular to the ecliptic plane, the axis $O_E Y_S$ complements the system to the right. We denote the unit vectors of this system as $\vec{e}_{X_S}$, $\vec{e}_{Y_S}$, $\vec{e}_{Z_S}$ respectively.

The position of the satellite in the ICS is determined by Euler angles: Ω is the longitude of the ascending node, i is the inclination of the orbit, u is the argument of latitude (Fig. 1).

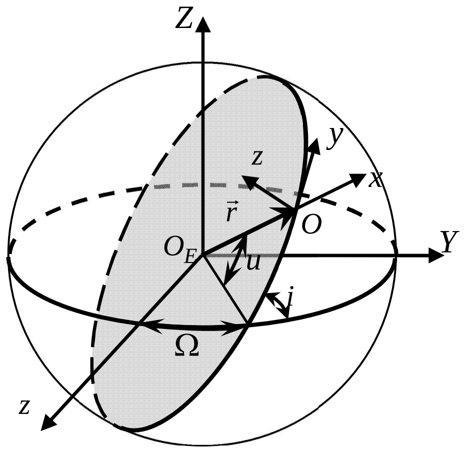


Fig. 1

The transition matrix from the ICS to the OCS has the form

$$T_{ou}=\begin{vmatrix} -\sin u\cos i\sin\Omega+\cos u\cos\Omega & \sin u\cos i\cos\Omega+\cos u\sin\Omega & \sin u\sin i \\ -\cos u\cos i\sin\Omega-\sin u\cos\Omega & \cos u\cos i\cos\Omega-\sin u\sin\Omega & \cos u\sin i \\ \sin i\sin\Omega & -\sin i\cos\Omega & \cos i \end{vmatrix}.$$

Equations of motion. To describe the motion, we will use a special form of equations for orbits close to circular [11, 12]. This form of equations describes the deviation of the satellite's trajectory from a circular unperturbed orbit. To do this, dimensionless variables b_1, b_2, γ associated with the current position and speed of the satellite are introduced

$$r=R_0(1+b_1),\quad \dot r=b_2\sqrt{\mu/R_0}\,,\quad p=R_0(1+\gamma);$$

where r is the distance from the center of the Earth to the satellite; R_0 is radius of the unperturbed circular reference orbit; $\dot r$ is radial speed of the satellite; μ is gravitational constant of the Earth; p is focal orbital parameter.

The equations of motion in such variables can be written in the form [16]

$$\begin{gathered} i'=zw\cos uF_n^*,\quad \Omega'=zw\frac{\sin u}{\sin i}F_n^*,\quad \Delta u'=w\left(\frac{s^{1/2}}{z^2}-1\right)-\Omega'\cos i, \\ b_1'=wb_2,\quad b_2'=w\frac{\gamma-b_1}{z^3}+wF_r^*,\qquad \gamma'=2wzsF_\tau^*, \end{gathered} \tag{1}$$

where $z=1+b_1$, $s=1+\gamma$, $w=\left(\dfrac{s^{1/2}}{z^2}-ctgi\sin uF_n^*\right)^{-1}$, $F_{\tau,n}^*=\dfrac{R_0^2}{\mu}s^{-\frac{1}{2}}F_{\tau,n}$, $F_r^*=\dfrac{R_0^2}{\mu}F_r$, F_r, F_τ, F_n are radial, transversal and normal disturbing accelerations, respectively; the prime indicates the derivative with respect to u; $\Delta u=u-\tilde u$, $\tilde u$ is latitude argument of unperturbed orbit, $\dot{\tilde u}=\sqrt{\dfrac{\mu}{R_0^3}}$; $\dfrac{\mu}{R_0^2}$ is the acceleration due to gravity for a given height.

Model of acceleration of solar radiation pressure forces. The vector of disturbing acceleration caused by solar radiation pressure (SRP) is determined as follows [10]

$$\vec F_{SRP}=-\frac{p_{SRP}c_RA_{SRP}}{m}\frac{\vec r_{Sun\,A}}{\left|\vec r_{Sun\,A}\right|},$$

where p_{SRP} is the force of solar radiation pressure per unit area (can be taken $p_{SRP}=4{,}57\times10^{-6}$ N/m^2); c_R is reflection coefficient; A_{SRP} is the surface area of the satellite affected by the sun's rays (illuminated part); m is mass of the satellite; $\vec r_{Sun\,A}$ is radius vector of the Sun relative to the satellite. We assume that the area A_{SRP} and coefficient c_R are constant.

Let us introduce the designation $\varepsilon_{SRP} = \dfrac{p_{SRP} c_R A_{SRP}}{m}$. Then, for example, for a satellite with a mass of 1 kg and an area of the illuminated part of the satellite of 1 m^2, the value of $\varepsilon_{SRP} \approx 9{,}1\times10^{-6}$ m/s^2 is a small value (the acceleration from the second zonal harmonic of the geopotential for the considered orbits is more than two orders of magnitude greater than this acceleration).

Taking into account the smallness ε_{SRP} and smallness of the ratio of the orbital radius to the distance to the Sun, it is not difficult to obtain that, with great accuracy, the acceleration caused by solar radiation pressure can be represented in the form

$$\vec{F}_{SRP} \approx -\varepsilon_{SRP} \vec{e}_S ,$$

where $\vec{e}_S$ is the unit vector of direction from the center of the Earth to the Sun.

First approximation equations. Since for the considered almost circular orbits b_1, b_2, γ are small values, and the disturbing acceleration from solar radiation pressure is small, then, neglecting values of the second order of smallness, in the first approximation, the equations of motion (1) can be represented in the form

$$\begin{aligned} &i' = \cos u F_n^*, \quad \Omega' = \frac{\sin u}{\sin i} F_n^*, \quad \Delta u' = \frac{1}{2}\gamma - 2b_1 - \Omega' \cos i, \\ &b_1' = b_2, \quad b_2' = \gamma - b_1 + F_r^*, \qquad \gamma' = 2F_\tau^*. \end{aligned} \tag{2}$$

Regularities of short-term changes in orbital parameters. To study the regularities of effects of solar radiation pressure forces and short-term (several days) changes in orbital parameters, we assume that the Sun is stationary in the ICS. Then the $O_E X_S Y_S Z_S$ coordinate system can be considered as inertial. The motion of the satellite in this coordinate system will be described by variables similar to those introduced earlier with the addition of the index “s”: b_{1S}, b_{2S}, γ_S, i_S, Ω_S, u_S. The equations of motion of the satellite (1), (2) will retain their form, only the index “s” will be added to the variables.

The position of the Sun in such a coordinate system is given by the ort of this system

$$\vec{e}_S = \vec{e}_{X_S} .$$

Using the transition matrix from the ICS to the OCS, we write the vector $\vec{e}_S$ in the form

$$\begin{aligned} \vec{e}_S = &\left[-\sin u_S \cos i_S \sin\Omega_S + \cos u_S \cos\Omega_S\right]\vec{e}_r + \\ &+\left[-\cos u_S \cos i_S \sin\Omega_S - \sin u_S \cos\Omega_S\right]\vec{e}_\tau + \sin i_S \sin\Omega_S \vec{e}_n . \end{aligned}$$

Substituting perturbing accelerations into equation (2), we can write

$$\begin{aligned} i_S' &= -\varepsilon(\vec{e}_S\cdot\vec{e}_n)\cos u_S = -\varepsilon \sin i_S \sin\Omega_S \cos u_S, \\ \Omega_S' &= -\varepsilon(\vec{e}_S\cdot\vec{e}_n)\frac{\sin u_S}{\sin i_S} = -\varepsilon\sin\Omega_S \sin u_S, \\ \gamma_S' &= -2\varepsilon(\vec{e}_S\cdot\vec{e}_\tau) = 2\varepsilon\left(\cos u_S \cos i_S \sin\Omega_S + \sin u_S \cos\Omega_S\right), \\ b_{1S}'' + b_{1S} &= \gamma_S - \varepsilon(\vec{e}_S\cdot\vec{e}_r) = \gamma_S - \varepsilon\left(-\sin u_S \cos i_S \sin\Omega_S + \cos u_S \cos\Omega_S\right), \end{aligned} \tag{3}$$

where $\varepsilon = \varepsilon_{SRP} \dfrac{R_0^2}{\mu}$ is a dimensionless quantity equal to the ratio of the acceleration of the satellite by solar radiation pressure to the acceleration of gravity at the height in question.

Note that neither the focal parameter (the modulus of the angular momentum) nor the shape of the orbit depend on the inertial coordinate system in which the motion is described. Therefore, in what follows we will remove the index "s" from b_{1s}, b_{2s}, γ_s.

Integrating the first three equations (they do not depend on b_1) with respect to u_S and maintaining the accepted accuracy, we obtain

$$\begin{gathered}\Delta i_S = -\varepsilon \sin i_S \sin\Omega_S \left(\sin u_S - \sin u_{S0}\right),\\ \Delta\Omega_S = \varepsilon \sin\Omega_S \left(\cos u_S - \cos u_{S0}\right),\\ \Delta\gamma = -2\varepsilon\left[\cos\Omega_S \left(\cos u_S - \cos u_{S0}\right) - \cos i_S \sin\Omega_S \left(\sin u_S - \sin u_{S0}\right)\right],\end{gathered} \tag{4}$$

where the sign Δ indicates a change in the orbital parameter under the effect of solar radiation pressure; index "0" hereinafter denotes the initial values of the variables, i.e. u_{S0} is the initial value of the argument of latitude u_S.

Let us write (4) in the form

$$\begin{gathered}\Delta i_S = -\varepsilon\left(\vec{e}_S \cdot \vec{e}_n\right)\left(\sin u_S - \sin u_{S0}\right),\\ \Delta\Omega_S = \varepsilon \frac{\left(\vec{e}_S \cdot \vec{e}_n\right)}{\sin i_S}\left(\cos u_S - \cos u_{S0}\right),\\ \Delta\gamma = -2\varepsilon\left[\left(\vec{e}_S \cdot \vec{e}_r\right) - \left(\vec{e}_S \cdot \vec{e}_{r0}\right)\right].\end{gathered} \tag{5}$$

From (4), (5) it is clear that the change in orbital parameters can be represented in the form

$$\Delta x = C\left(u_{s0}\right) + B\cos\left(u_S - \varphi\right),$$

where x is the orbital parameter, C is a constant determined by the initial conditions, B, φ are amplitude and phase shift. That is, the change in orbital parameters to a first approximation is described by harmonic oscillations, and is determined by the amplitude and phase shift of these oscillations.

Changes in orbital plane orientation. From (5) it follows that the amplitude of forced oscillations of the angle of inclination i_S to the ecliptic plane is proportional to the cosine of the angle between the direction to the Sun and the normal to the orbital plane. The phase shift of these oscillations is $\pi/2$ at $(\vec{e}_S \cdot \vec{e}_n) < 0$ and $-\pi/2$ at $(\vec{e}_S \cdot \vec{e}_n) > 0$.

From (4) it follows that the amplitude of forced oscillations of the longitude of the ascending node in the ecliptic plane Ω_S is proportional to $\sin\Omega_S$, i.e. sine of the angle between the direction to the Sun and the line of nodes (cosine of the angle between the line of nodes and the axis $O_E Y_S$). The phase shift of these oscillations is zero.

To understand the physics (mechanics) of the process that changes the orientation of the orbital plane, it is necessary to introduce the following axes into the orbital plane: the $O_E N_S$ axis, directed along the line of nodes to the ascending node of the orbit, and the $O_E P_S$ axis perpendicular to the $O_E N_S$ and directed towards the movement of the satellite in the ascending node. That is, the direction of the $O_E N_S$ and $O_E P_S$ axes coincides with the direction of the Ox and Oy axes in the ascending node.

The moment of solar radiation pressure forces relative to the $O_E N_S$ (line of nodes) is aimed at changing the orbital inclination. At points $u_S = \pm\pi/2$ this moment reaches a maximum (minimum), and at points $u_S = 0, \pi$ it is zero. That is, on a half-turn of the satellite $u_S \in [0, \pi]$, this moment tends to rotate the orbital plane relative to the $O_E N_S$ in one direction, and on the other half-turn $u_S \in [\pi, 2\pi]$, in the other direction.

The moment of solar radiation pressure forces relative to the $O_E P_S$ is aimed at changing the longitude of the ascending node. At points $u_S = 0, \pi$ this moment reaches a maximum (minimum), and at points $u_S = \pm\pi/2$ it is zero. That is, on a half-turn of the satellite $u_S \in [-\pi/2, \pi/2]$, this moment tends to rotate the orbital plane relative to the $O_E P_S$ in one direction, and on the other half-turn $u_S \in [\pi/2, 3\pi/2]$, in the other direction.

The change in the orientation of the orbital plane (the orientation of the angular momentum of orbital motion) is determined by the action of the gyroscopic moment (the moment of two rotations). This moment balances the impact of the moment of external forces aimed at changing the orientation (see [12]). Therefore, the change in orientation is perpendicular to the direction of the applied moment of external forces. The moment of solar radiation pressure forces relative to $O_E P_S$, aimed at changing the longitude of the ascending node, is compensated by the change (rate of change) in the orbital inclination. And the moment of solar radiation pressure forces relative to $O_E N_S$, aimed at changing the inclination of the orbit, leads to a change in the longitude of the ascending node.

Changing the focal orbital parameter. The amplitude of forced oscillations of the focal orbital parameter is proportional to the cosine of the angle between the direction of the sun's rays (opposite to the direction to the Sun) and the direction to the satellite ($\vec{r}$). The phase shift of these oscillations is φ, where

$$\cos\varphi = -\cos\Omega_S/q,\ \sin\varphi = \cos i_S \sin\Omega_S/q,\ q = \sqrt{\cos^2\Omega_S + \cos^2 i_S \sin^2\Omega_S}\ . \quad (6)$$

That is,

$$(\vec{e}_S \cdot \vec{e}_r) = -q\cos(u_S - \varphi) \text{ and } (\vec{e}_S \cdot \vec{e}_\tau) = -q\cos(u_S + \pi/2 - \varphi)\ .$$

It is easy to see that this angle φ corresponds to the maximum angle of deviation $\vec{r}$ from the direction towards the Sun ($\vec{e}_S$). The moment of solar radiation pressure forces relative to the binormal to the orbit is proportional to $-(\vec{e}_S \cdot \vec{e}_\tau)$, and reaches a minimum at point $u_S = \varphi + \pi/2$, and a maximum at point $u_S = \varphi + 3\pi/2$. That is, on half a revolution of the satellite $u_S \in [\varphi, \varphi + \pi]$ this moment reduces the angular momentum of the satellite, and on the other half turn $u_S \in [\varphi + \pi, \varphi + 2\pi]$ it increases.

The change in the shape of the orbit is described by the parameter b_1. To determine the main regularities of these changes, one can proceed in two ways. In the first, we consider the changes as changes in the oscillations of a linear oscillator during forced oscillations. Then the initial equation for analyzing motion is

$$b_1'' + b_1 = \gamma_0 - 2\varepsilon\left[(\vec{e}_S \cdot \vec{e}_r) - (\vec{e}_S \cdot \vec{e}_{r0})\right] - \varepsilon(\vec{e}_S \cdot \vec{e}_r) . \quad (7)$$

Using the notation (6) introduced when analyzing changes in γ_S, we can write (7) in the form

$$b_1'' + b_1 = \tilde{b}_1 + 3\varepsilon q \cos(u_S - \varphi), \quad (8)$$

where $\tilde{b}_1$ is the constant displacement b_1,

$$\tilde{b}_1 = \gamma_0 + 2\varepsilon\left[(\vec{e}_S \cdot \vec{e}_{r0})\right].$$

Consequently, the change in the shape of the orbit under the effect of solar radiation pressure is described in a first approximation by the equations of a linear oscillator under a resonant periodic perturbation. The general solution to equation (8) can be represented as

$$\begin{aligned} b_1 &= \tilde{b}_1 + A\cos(u_S - \alpha) + \frac{3}{2}\varepsilon q\left[\sin\varphi \sin(u_S - u_{S0}) + (u_S - u_{S0})\cos\left(u_S - \varphi - \frac{\pi}{2}\right)\right] = \\ &= \tilde{b}_1 + A\cos(u_S - \alpha) - \frac{3}{2}\varepsilon\left[\cos i_S \sin\Omega_S \sin(u_S - u_{S0}) - (u_S - u_{S0})(\vec{e}_\tau \cdot \vec{e}_S)\right], \end{aligned} \quad (9)$$

where the quantity $A\cos(u_S - \alpha)$ denotes the natural oscillations of the orbital radius.

Thus, the main effect of solar radiation pressure forces is the excitation of forced oscillations of the orbital radius with amplitude linearly increasing in time. The phase shift of these oscillations is equal to $\varphi + \pi/2$, which corresponds to the minimum angle between the transversal to the orbit and the direction to the Sun. Consequently, the maximum (apogee) of these oscillations is located at the point where light pressure forces maximally slow down the motion of the satellite (directed against the speed), and the minimum (perigee) is at the point of maximum acceleration of motion.

From an energy point of view, solar radiation pressure forces do not change the energy of rotational motion, but pump energy into the oscillations of the satellite relative to the original orbit. Of course, this applies only to the effect of solar radiation pressure forces on motion in almost circular orbits, considered in a first approximation. A linear increase in the deformation of the orbital shape will lead to a violation of the symmetry of the effect of solar rays on half-rotations of the satellite and new effects in motion. Consideration of this issue is beyond the scope of the problem under consideration and requires additional research.

Note that the considered regularities of changes in orbital parameters make it possible to judge the effect of the shadow on the motion of the satellite. It is easy to imagine how the shadow from the Earth breaks the symmetry of the action of the moment of forces relative to $O_E P_S$, aimed at changing the longitude of the ascending node Ω_S. In this case, the effect of solar radiation pressure forces will lead to a systemic change in the inclination of the orbit to the ecliptic plane (possibly increasing and decreasing) i_S. And it is difficult to imagine that the shadow from the Earth (sphere) violates the symmetry of the action of the moment relative to the line of nodes due to the symmetry of the shadow relative to the ecliptic plane. Consequently, the shadowing of part of the orbit can only lead to a decrease in the amplitude of forced oscillations of the angle Ω_S. Of course, the longitude of the ascending node Ω in the equatorial plane may have systemic changes due to changes in i_S.

The second, most commonly used way to study changes in the shape of an orbit is to fix its shape and consider changes in the parameters of this shape. Let's assume that

$$b_1 = \tilde{b}_1 + A\cos(u_S - \alpha), \quad b_2 = -A\sin(u_S - \alpha), \tag{10}$$

and A, and α will be considered as new variables.

After transformations, it is easy to obtain [16] that the equations for changing A and α can be written in the form

$$A' = -\left(\Delta_p\gamma + F_r^*\right)\sin(u_S - \alpha), \quad A\alpha' = \left(\Delta_p\gamma + F_r^*\right)\cos(u_S - \alpha),$$

where $\Delta_p\gamma$ are periodic changes in the value of $\Delta\gamma$ under the effect of solar radiation pressure.

Then

$$\begin{aligned} A' &= 3\varepsilon(\vec{e}_r \cdot \vec{e}_S)\sin(u_S - \alpha) = -3\varepsilon q\cos(u_S - \varphi)\sin(u_S - \alpha), \\ A\alpha' &= -3\varepsilon(\vec{e}_r \cdot \vec{e}_S)\cos(u_S - \alpha) = 3\varepsilon q\cos(u_S - \varphi)\cos(u_S - \alpha). \end{aligned} \tag{11}$$

From (11) we can write

$$\begin{aligned} A' &= -\frac{3}{2}\varepsilon q\left[\sin(\varphi - \alpha) + \cos(\varphi + \alpha)\sin 2u_S - \sin(\varphi + \alpha)\cos 2u_S\right], \\ A\alpha' &= \frac{3}{2}\varepsilon q\left[\cos(\varphi - \alpha) + \cos(\varphi + \alpha)\cos 2u_S + \sin(\varphi + \alpha)\sin 2u_S\right]. \end{aligned} \tag{12}$$

Or averaging (12) over u_S

$$\begin{aligned} \overline{A}' &= -\frac{3}{2}\varepsilon\left(\cos i_S \sin\Omega_S \cos\overline{\alpha} + \cos\Omega_S \sin\overline{\alpha}\right) = -\frac{3}{2}\varepsilon q\sin(\varphi - \overline{\alpha}), \\ \overline{A}\overline{\alpha}' &= \frac{3}{2}\varepsilon\left(\cos i_S \sin\Omega_S \sin\overline{\alpha} - \cos\Omega_S \cos\overline{\alpha}\right) = \frac{3}{2}\varepsilon q\cos(\varphi - \overline{\alpha}), \end{aligned} \tag{13}$$

where $\overline{A}, \overline{\alpha}$ are averaged values.

Integrating (12), to a first approximation we obtain

$$\begin{aligned} \Delta A &= -\frac{3}{2}\varepsilon q\left[\sin(\varphi - \alpha)(u_S - u_{S0}) - \frac{1}{2}\cos(2u_S - \varphi - \alpha) + \frac{1}{2}\cos(2u_{S0} - \varphi - \alpha)\right], \\ \Delta\alpha &= \frac{3}{2}\varepsilon\frac{q}{A}\left[\cos(\varphi - \alpha)(u_S - u_{S0}) + \frac{1}{2}\sin(2u_S - \varphi - \alpha) - \frac{1}{2}\sin(2u_{S0} - \varphi - \alpha)\right]. \end{aligned} \tag{14}$$

By substituting (14) into (10), we can verify that solutions (14) correspond to solution (9) to a first approximation. From (13), (14) it follows that the phase shift of natural oscillations α tends to $\varphi + \pi/2$, and their amplitude tends to grow linearly. At the same time, the physical interpretation of the results obtained with this research approach is significantly difficult. Fixing the oscillation shape directs research to search for distortions of this shape, and thereby complicates (distorts) the search for regularities of movement.

Accounting for the movement of the Sun. Changing parameters in the equatorial ICS. Taking into account the movement of the Sun in the first approximation under consideration will lead to the appearance on the right sides of the equations of slowly changing parameters that describe the position of the Sun in the ICS: r_S, λ, δ are distances to the Sun, longitude and declination of the Sun, respectively.

The unit vector of the Sun in the ICS is given by the expression

$$\vec{e}_S = \cos\delta\cos\lambda\,\vec{e}_X + \cos\delta\sin\lambda\,\vec{e}_Y + \sin\delta\,\vec{e}_Z .$$

In the OCS, the $\vec{e}_S$ vector can be written in the form

$$\begin{aligned}\vec{e}_S = &\left[\cos\delta\left(\cos u\cos\beta - \sin u\cos i\sin\beta\right) + \sin u\sin i\sin\delta\right]\vec{e}_r + \\ &+\left[\cos\delta\left(-\sin u\cos\beta - \cos u\cos i\sin\beta\right) + \cos u\sin i\sin\delta\right]\vec{e}_\tau + \\ &+\left(\sin i\cos\delta\sin\beta + \cos i\sin\delta\right)\vec{e}_n ,\end{aligned}$$

where $\beta = \Omega - \lambda$.

It is not difficult to understand that the first equalities of equations (3) will also be preserved for variables describing the motion of the satellite in the ICS (for variables, the index s must be removed). Then, integrating these equations to a first approximation, we obtain equalities similar to (5)

$$\begin{aligned}\Delta i &= -\varepsilon(\vec{e}_S\cdot\vec{e}_n)\left(\sin u - \sin u_0\right),\\ \Delta\Omega &= \varepsilon\frac{(\vec{e}_S\cdot\vec{e}_n)}{\sin i}\left(\cos u - \cos u_0\right),\\ \Delta\gamma &= -2\varepsilon\left[(\vec{e}_S\cdot\vec{e}_r) - (\vec{e}_S\cdot\vec{e}_{r0})\right].\end{aligned} \tag{15}$$

Only the expressions of the direction cosines $\vec{e}_S$ in the OCS will be more cumbersome.

The above-described regularities of changes in angles i_S, Ω_S will not change even taking into account the movement of the system $O_E X_S Y_S Z_S$, and changes in angles i, Ω can be expressed through changes in angles i_S, Ω_S.

The regularities of changes in γ will not change either. Only the expressions for q and φ will change

$$\begin{aligned}&\cos\varphi = -\cos\delta\cos\beta / q,\ \ \sin\varphi = (\cos\delta\cos i\sin\beta - \sin i\sin\delta)/q,\\ &q = \sqrt{\cos^2\delta\cos^2\beta + (\cos\delta\cos i\sin\beta - \sin i\sin\delta)^2}.\end{aligned} \tag{16}$$

As before,

$$(\vec{e}_S\cdot\vec{e}_r) = -q\cos(u-\varphi) \text{ and } (\vec{e}_S\cdot\vec{e}_\tau) = -q\cos(u+\pi/2-\varphi).$$

The regularities of changes in the shape of the orbit will also remain. Equations (7), (8), the first part of equality (9), equations (11)–(14) will remain unchanged, only the index "s" must be removed, and q and φ are determined by equality (16), not equality (6).

At the same time, the movement of the Sun relative to the satellite's orbit leads to long-period changes and can qualitatively change the picture of the evolution of parameters.

Let us consider the case of a sun-synchronous orbit "sunrise-sunset", which is important for remote sensing satellites. Such orbits are often used by Earth radar observation satellites to maintain constant illumination of the satellite's solar panels. In this case, the angle β as a first approximation can be considered constant and close to ±90°.

Let us assume that $\beta = \pi/2 + \Delta\beta$, where $\Delta\beta << 1$. Then, to a first approximation in small quantities

$$\Delta i = -\varepsilon \sin(i+\delta)\sin u, \qquad \Delta\Omega = \varepsilon \frac{1}{\sin i}\sin(i+\delta)(\cos u - 1),$$

where it is accepted that $u_0 = 0$.

When $i + \delta \approx \pi/2$, i.e. $|\cos(i+\delta)| << 1$, then $q << 1$, and to a first approximation, solar pressure does not change the shape of the orbit, and the amplitude of oscillations of the focal parameter is of the second order of smallness. This is understandable, since in this case the direction to the Sun is almost perpendicular to the orbital plane.

When $|\cos(i+\delta)|$ is not small, $q \approx |\cos(i+\delta)|$ and $\phi \approx sign[\cos(i+\delta)]\pi/2$. Then, to a first approximation, for $u_0 = 0$

$$\Delta\gamma = 2\varepsilon\cos(i+\delta)\sin u,$$

$$\Delta A = -\varepsilon\frac{3}{4}\cos(i+\delta)[\sin\alpha - 2u\cos\alpha + \sin(2u-\alpha)],$$

$$\Delta\alpha = \varepsilon\frac{3}{4}\frac{1}{A}\cos(i+\delta)[\cos\alpha + 2u\sin\alpha - \cos(2u-\alpha)].$$

Thus, for low solar synchronous orbits ($i \approx 96°$), the annual movement of the Sun's declination leads to a change in the direction of evolution of the orbital shape to the opposite.

Numerical estimates. Calculations showed good agreement between the solutions of differential equations of motion (1) and analytical solutions (14), (15) at the initial stage (Fig. 2).

The figures show the change in orbital parameters due to the action of solar radiation pressure during one day under the following initial conditions: date – September 4, 2023 3:42:50 UTC, $\beta \approx 95°$, $\delta \approx 7{,}4°$, $R_0 \approx 6882$ km (orbital altitude is approximately 511 km), $i \approx 97{,}4°$, $\Omega \approx 257{,}5°$, $\gamma \approx 0{,}0007$, $A \approx 0{,}00119$, $\alpha \approx 281{,}1°$ (which corresponds to $b_1 \approx 0{,}00023$, $b_2 \approx -0{,}00117$), $\varepsilon_{SRP} \approx 8{,}835 \times 10^{-8}$ m/c^2.

The effect of solar radiation pressure on changes in orbital parameters is determined by the value $\varepsilon = \varepsilon_{SRP}\frac{R_0^2}{\mu} = \frac{p_{SRP}c_R A_{SRP}}{m}\frac{R_0^2}{\mu}$, which for a satellite with an area $A_{SRP} = 1$ m^2 and a mass $m = 100$ kg at altitudes of 250 km – 650 km will vary from approximately $1{,}01 \times 10^{-8}$ to $1{,}13 \times 10^{-8}$, which corresponds to the maximum amplitude of inclination fluctuations due to the action of solar radiation pressure of the order of $0{,}65 \times 10^{-6}$ degrees.

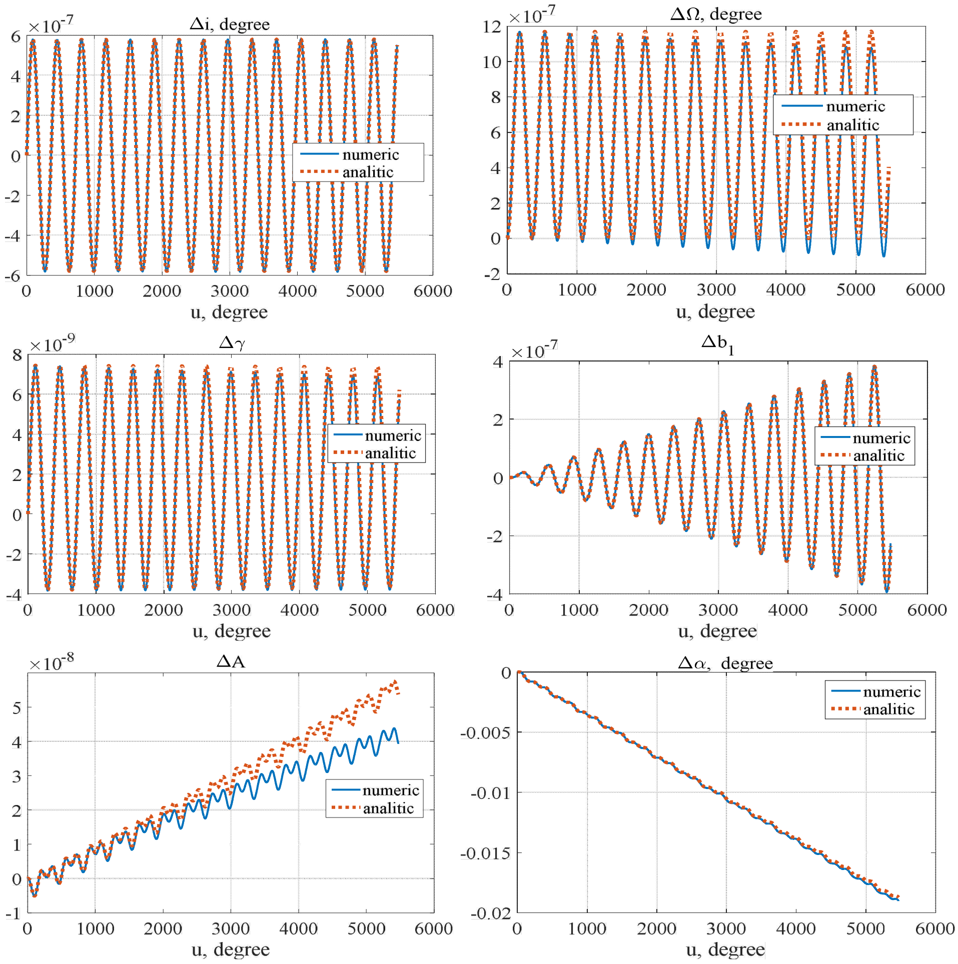
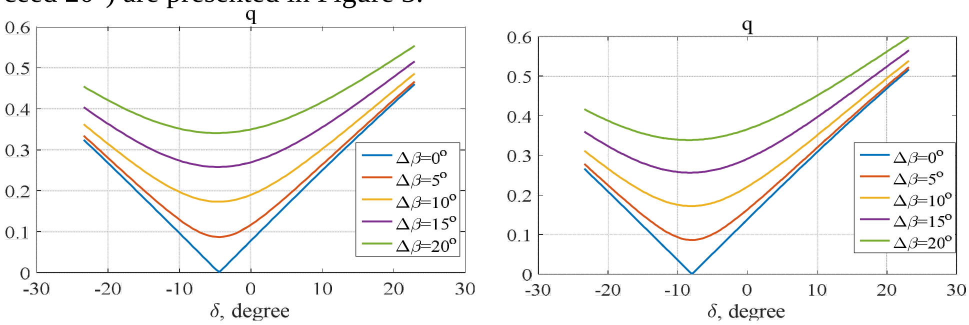


Fig. 2

The increase in the amplitude of the change in the radius of the orbit is determined by the value q , which is an even function relative to $\Delta\beta$. Possible changes in q for the dawn-dusk SSO (it is assumed that for such orbits $\Delta\beta$ does not exceed 20°) are presented in Figure 3.



a) b)

Fig. 3

Fig. 3, a) shows the change in q for an orbital altitude of 250 km, for which the SSO inclination is approximately 94,47°; Fig. 3, b) – for an orbital altitude of

650 km, for which the SSO inclination is approximately 97,96°. As we can see, the maximum value of q under the accepted assumptions will not exceed 0,6. Consequently, for a satellite with an area A_{SRP} =1 m^2 and a mass m =100 kg at low SSO, the maximum possible increase in the amplitude of change in the orbital radius (value b_1) per one orbital revolution will not exceed a value of $3\pi\varepsilon q \approx 6{,}39\times10^{-8}$, which corresponds to a change in the radius at an altitude of 650 km of the order of 0,45 m per revolution.

Conclusions.

1. Simple analytical expressions have been constructed to describe the main regularities of short-term (several days) changes in orbital parameters. It is shown that the change in the orientation of the orbital plane is determined by the action of the gyroscopic moment. This moment balances the effect of the moment of external forces aimed at changing orientation, and the change in orientation is perpendicular to the direction of the applied moment of external forces. The main effect of solar radiation pressure forces is the excitation of forced oscillations of the orbital radius with amplitude linearly increasing with time. The maximums of these oscillations (apogee) is at the point where the light pressure forces maximally slow down the motion of the satellite (directed oppositely to the velocity), and the minimums (perigee) is at the point of the maximum motion acceleration.

2. It is shown that the annual movement of the Sun can qualitatively change the picture of the evolution of orbital parameters. For sun-synchronous dawn-dusk orbits, compact analytical solutions for changes in orbital parameters are constructed, and it is shown that the annual movement of the Sun's declination reverses the direction of evolution of the orbital shape.

3. The calculations showed a reasonably high accuracy of the analytical solutions at the initial stage. The obtained numerical estimates make it possible to evaluate the effect of the solar pressure on changes in orbital parameters.

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