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MATHEMATICAL MODELING OF DETERMINING THE KINETIC PARAMETERS OF CHARGED PLASMA PARTICLES USING AN INSULATED PROBE SYSTEM IN THE IONOSPHERIC CONDITIONS

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Метою статті є теоретичне обґрунтування можливості визначення кінетичних параметрів заряджених частинок іоносферної плазми за окремими вимірами струмів ізольованої зондової системи в області насичення електронів.

При проведенні досліджень використано методи фізичного моделювання, числового розв'язання систем нелінійних диференціальних рівнянь, аналізу невизначеності вимірювань та комп'ютерного моделювання.

Зондова система складається з циліндричних електродів – зонда та опорного електрода. Відношення площ опорного електрода і зонда може бути значно меншим за вимоги теорії одиночного циліндричного зонда. Електроди поперечно обтікаються надзвуковим вільномолекулярним потоком плазми.

Склад заряджених частинок іоносферної плазми моделюється позитивними іонами атомарного кисню, атомарного водню та електронами, що забезпечують квазінейтральність плазми. Разом із математичною моделлю плазми з двосортними іонами розглянута модель плазми з модельними односортними іонами, маса яких вибирається так, щоб іонний струм насичення на циліндр був однаковим для обох моделей. На основі отриманого раніше асимптотичного розв'язку для струму насичення електронів у плазмі з односортними іонами отримано співвідношення, що зв'язує комплекс кінетичних параметрів заряджених частинок (температуру, спрямовану швидкість іонів та температуру електронів) плазми з результатами вимірювань зондових струмів. Числове та аналітичне дослідження цього співвідношення в рамках математичної моделі плазми з двосортними іонами дозволило знайти розрахункові формули для визначення кінетичних параметрів заряджених частинок за окремими вимірюваннями струмів ізольованої зондової системи в області насичення електронів.

Отримано числові та аналітичні оцінки похибок розрахункових формул при визначенні кінетичних параметрів плазми з двосортними іонами залежно від відношення площин поверхонь електродів зондової системи, потенціалів зсуву зонда щодо опорного електрода та точності вимірювання зондових струмів.

Визначено діапазони параметрів зондової системи, що забезпечують максимальну достовірність вимірювань в умовах іоносфери.

Ключові слова: *беззіштовхувальна плазма, зондова система з циліндричними електродами, моделі плазми з двосортними та односортними іонами, математичні моделі збирання струму, швидкість спрямованого руху іонів, температури заряджених частинок.*

The goal of this article is to theoretically substantiate the possibility of determining the kinetic parameters of charged particles of the ionospheric plasma by measuring the currents of an insulated probe system in the electron saturation region.

Methods of physical modeling, numerical integration of nonlinear differential equations, measurement uncertainty analysis, and computer modeling were used.

The probe system consists of cylindrical electrodes: a probe and a reference electrode. The ratio of the reference electrode and the probe areas can be significantly smaller than required by the single cylindrical probe theory. The electrodes are placed transversely in a supersonic free-molecular plasma flow.

The charged particle composition of the ionospheric plasma is modeled by positive ions of atomic oxygen and atomic hydrogen and by electrons, which ensure plasma quasi-neutrality. Along with a mathematical model of plasma with two ion species, a model of a one-component plasma is considered with the ion mass selected so that the ion saturation current to the cylinder may be the same for both models. Based on an earlier asymptotic solution for the electron saturation current in a one-component plasma, the kinetic parameters of charged particles (the ion temperature and directed velocity and the electron temperature) were related to the measured probe currents. A numerical and an analytical study of this relationship within the framework of the mathematical model of a plasma with two ion species resulted in analytical expressions for determining the kinetic parameters of charged particles from the measured currents of the insulated probe system in the electron saturation region.

The errors of the analytical expressions in determining the kinetic parameters of a plasma with two ion species were estimated numerically and analytically as a function of the probe system's electrode area ratio and the probe current measurement accuracy.

The ranges of the probe system parameters that maximize the measurement reliability in the ionospheric conditions were determined.

Keywords*: collisionless plasma, probe system with cylindrical electrodes, plasma models with one and two ion species, mathematical models of current collection, directed velocity of ions, temperatures of charged particles.*

Introduction. The simple equipment and acceptable accuracy of determining the local plasma parameters make stationary cylindrical Langmuir probe a perspective tool for ionospheric monitoring [1 – 5].

The spacecraft body which is conductive and not insulated from the plasma is usually used as a reference electrode for the electric probe. Mounting a single Langmuir probe on ultra-small satellites imposes conflicting restrictions. For the reliable diagnostics of a highly rarefied ionospheric plasma the area of the probe's current-collecting surface must be large. On the other hand, a sufficiently strict condition $S_s = S_{cp}/S_p \geq 10^4$ for the ratio of the areas of the reference electrode S_{cp} and the electric probe S_p at a relatively small area of the satellite's conductive outer surface contacting the plasma restricts the probe area. In such a situation, we make use of the measuring probe system that is not electrically connected to the spacecraft body [6].

An analysis of the available experimental data [7 – 9] shows that in many cases at altitudes above 300 km the ionospheric plasma can be approximately considered as a weakly ionized gas mixture, the charged components of which are electrons and ions of two species with significantly different masses – atomic ions of hydrogen and oxygen.

The article [9] provides a theoretical substantiation of the possibility of determining the charged particles density in the plasma with two ions species based on current measurements by the insulated probe system (IPS) with cylindrical electrodes oriented transversely in a supersonic flow. This article is a continuation of the work [9]. A theoretical substantiation is provided for the possibility of determining the kinetic parameters of charged particles in plasma with two ions species based on separate measurements of IPS currents in the electron saturation region. Calculation relationships are revealed, and the influence of the IPS's geometric parameters and the errors in measuring currents and voltages on the reliability of determining the flow velocity and temperatures of charged particles is estimated.

Problem formulation. Let's consider the problem of collecting currents by the IPS, cylindrical electrodes of which are oriented transversely to the bulk velocity in a supersonic flow of a weakly ionized gas mixture, the charged particles of which are electrons and singly charged atomic ions of oxygen O^+ and hydrogen H^+. From the probe measurements perspective, we neglect the effects of the interaction between charged and neutral components of the gas mixture and the influence of the magnetic field. The unperturbed plasma is considered to be Maxwellian, quasi-neutral, with equal ion temperatures $T_{H^+} = T_{O^+} = T_i$. The ion composition of the plasma is characterized by the parameter $\chi_n = n_{H^+}/(n_{H^+} + n_{O^+}) \equiv n_{H^+}/n_e$, where n stands for density, index e – for electrons.

The probe system consists of a measuring electrode (probe) with a base radius of r_p and a reference electrode with a base radius of r_{cp}. The electrodes are placed transversely in a supersonic plasma flow with the bulk velocity of V. The base radii of the electrodes must be significantly less than their lengths, and the end surfaces are insulated from the plasma. It is assumed that electrostatic and gas-dynamic influence of electrodes on each other is small, the emission current from electrode surfaces is absent, the flow around electrodes is free-molecular. The base radii of the electrodes must satisfy the conditions:

$$r_p/\lambda_d \leq 1 \ , \ \ r_{cp}/\lambda_d < \xi^* = 3\ldots10,$$

where λ_d is the Debye length in an unperturbed plasma, ξ^* is the value of r_{cp}/λ_d at which the Langmuir formula for the ion current on a cylinder is applicable [10].

The aim of this article is to develop a procedure for determination the mass velocity and temperature of ions, electron temperature based on the results of the currents measurements using the IPS in the electron saturation region.

Mathematical models of current collection in a flow of rarefied oxygen-hydrogen plasma and plasma with one-species model ions in the electron saturation region are developed in [9]. A mathematical model of current collection by a cylindrical electrode in supersonic free-molecular flow of plasma with ions of two species is based on the assumption that the presence of ions with different masses does not lead to a significant change in the self-consistent electric field in the vicinity of the electrode compared to the one-species plasma [8]. In dimensionless form, the total current on the cylinder with electric potential φ relative to the potential of the unperturbed plasma with oxygen and hydrogen ions, is estimated by the relationship [9] (the electron current is positive):

$$\bar{I}_c(\varphi) = \bar{I}_e(\varphi) - 4\chi_n\sqrt{\mu_2/\beta}\cdot\bar{I}_H(\varphi) - (1-\chi_n)\sqrt{\mu_2/\beta}\cdot\bar{I}_O(\varphi), \quad S_i > 4 \qquad (1)$$

where $\bar{I}_c$, $\bar{I}_e$ are the total and electron currents on the cylinder, respectively, normalized by the thermal electron current; $\bar{I}_H$, $\bar{I}_O$ are the currents of atomic hydrogen and oxygen ions on the cylinder, respectively, normalized to the thermal currents of ions of the corresponding species, $\varphi = eU/kT_e$ is the dimensionless electric potential (U is the dimensional potential), k is the Boltzmann constant, e is the unit charge, $\mu_2 = m_e/m_O$ is the ratio of the masses of electrons and atomic oxygen, $\beta = T_e/T_i$ is the ratio of the temperatures of charged particles, $S_i = V/u_{O^+}$ is the velocity ratio for oxygen ions. The thermal current of particles of sort α is $I_{\alpha,0} = j_\alpha \cdot S_c$, where $j_\alpha = en_\alpha u_\alpha/2\sqrt{\pi}$ is the density of the thermal current of particles, $u_\alpha = \sqrt{2kT_\alpha/m_\alpha}$ is the thermal velocity, T_α and m_α is the temperature and mass of the particles, S_c is the area of the collecting surface of the cylinder. Here and below, the index $\alpha = i$ refers the value to the ions, O – to atomic oxygen ions, H – to atomic hydrogen ions, e – to electrons. The calculation formulas for the currents $\bar{I}_e$, $\bar{I}_H$, $\bar{I}_O$ are presented in [9].

The current-voltage characteristic (CVC) of the probe $\bar{I}_p(\varphi_{iz})$, taking into account (1), is modeled by the system of nonlinear equations

$$\bar{I}_p(\varphi_{iz}) = \bar{I}_c(\varphi_{iz} + \varphi_{cp}), \qquad (2)$$

$$S_s \cdot \bar{I}_c(\varphi_{cp}) + \bar{I}_c(\varphi_{iz} + \varphi_{cp}) = 0, \qquad (3)$$

where φ_{iz} is the probe potential relative to the reference electrode (bias potential), φ_{cp} is the equilibrium potential of the reference electrode relative to the potential of the unperturbed plasma, which corresponds to the bias potential φ_{iz}, and S_s is the geometric parameter of the IPS.

In order to obtain simple relationships between the results of probe measurements and plasma parameters, a mathematical model of plasma with model single-species ions of mass $m_{mod} = (3\chi_n + 1)^{-2} \cdot m_O$ is proposed in [9]. The mass of these model ions is selected to ensure the equality of ion currents to the reference electrode in the model for two ions species (1) and single-species model ions. The main parameters (electron density, ion and electron temperatures, bulk flow velocity) and properties (quasineutrality, Maxwellian distribution of particles velocities) of plasma with model single-species ions remain the same as those of the considered oxygen-hydrogen plasma. For the CVC of the IPS, this mathematical model in the electron saturation region leads to a straightforward asymptotic relationship [8]:

$$\bar{I}_p(\varphi_{iz}) \approx \frac{2}{\sqrt{\pi}} \cdot \sqrt{\frac{S_s{}^2 \mu_{mod}}{1 + S_s{}^2 \mu_{mod}}} \cdot \sqrt{\left(1/2 + S_{mod}^2\right)/\beta + \pi/4 + \varphi_{iz}}\,, \tag{4}$$

where $\mu_{mod} = m_e / m_{mod}$ is the ratio of the charged particles masses, $S_{mod} = V / u_{mod}$ is the ion velocity ratio, $u_{mod} = \sqrt{2kT_i / m_{mod}}$ is the thermal velocity of the model ions.

The inverse problem is to determine the plasma parameters (parameters of the mathematical model of current collection (1) – (3)) using the results of measuring probe current $I_p(U_{iz})$ at specified geometric parameters of the IPS. It is known [8] that, unlike for the case of a single Langmuir probe, the electron saturation current of the IPS depends on the ion flow velocity S_i and the degree of plasma non-isothermality β. We use this circumstance to determine the kinetic parameters of charged plasma particles V, T_i, T_e based on the single-species ions model (4) in the electron saturation region, where the measured current significantly exceeds the probe current collected in the ion branch of the CVC.

Within the framework of the single-species model (4), the calculation relationships for determining the kinetic parameters of the model plasma can be obtained as a special case of the results of [11] where determining the kinetic parameters of charged particles of a gas-discharge source jet is described. The relationship obtained in [11] between the kinetic parameters of charged particles of a supersonic jet of dissociated diatomic gas and the IPS currents measured in the electron saturation region, is valid for any degree of ion dissociation η. Thus, assuming $\eta = 0$ and the ion mass $m_i = m_{mod}$, we obtain a relationship between the temperature and the bulk velocity of ions and the temperature of electrons in a supersonic flow of a model plasma and the probe current measured by IPS within the framework of the plasma model with single-species ions (4):

$$K = \frac{m_{\mathrm{mod}} V^2}{2e} + \frac{kT_i}{2e} + \frac{\pi k T_e}{4e} \approx \frac{I_p{}^2(U_{\mathrm{iz}}) \cdot dU}{I_p{}^2(U_{\mathrm{iz}} + dU) - I_p{}^2(U_{\mathrm{iz}})} - U_{\mathrm{iz}} = D_k\,, \tag{5}$$

where I_p is the dimensional probe current (in amps) of the IPS with the electrode areas ratio of S_s, dU is the increment of the dimensional bias potential U_{iz} (in volts). The bias potential U_{iz} must be within the range of the applicability of solution (4), [9]:

$$U_{iz} > S_s{}^2 \mu_{mod} \cdot K - \left(S_s{}^2 \mu_{mod} + 1 \right) \cdot \frac{\pi k T_e}{4e} . \tag{6}$$

In the left side of (5), the first term $m_{\text{mod}} V^2 / 2e$ characterizes the average kinetic energy of the bulk motion of ions (in eV), the second term $kT_i / 2e$ characterizes the average kinetic energy of the thermal motion of ions, and the third term $\pi k T_e / 4e$ characterizes the average kinetic energy of the thermal motion of electrons. Relation (5) can be used as a calculation formula for determining the kinetic parameters of a model plasma with ions of mass m_{mod}.

It follows from (6) that an increase in the ratio of the electrode areas S_s leads to an increase in the required values of the bias voltage U_{iz} in (5). Within the framework of model of single-species ions (4), relation (5) is sufficiently accurate [11]. Thus, for the ratio of the electrode areas $S_s \le 200$, relation (5) is satisfied with an error of less than 0.1% at $U_{iz} > 80$ V. However, according to the problem formulation, the currents and bias voltages of the IPS can only be measured in two-species ions plasma, i.e. within the framework of model (1) – (3).

Analysis of the result of the performed numerical modeling of D_K behavior using the model (1) – (3) at various ions composition χ_n and kinetic parameters of the plasma shows that as bias potential U_{iz} increases, D_K asymptotically approaches to a certain constant value, which is greater than the value of K calculated for a model plasma with single-species ions of mass m_{mod}.

Fig. 1 shows the dependence of D_K (in volts) on bias potential U_{iz} (in volts) calculated by the model (1) – (3) for various electrodes areas ratios S_s and increments of bias potential dU at χ_n = 0.5. The solid curves correspond to $dU = 10$ V, S_s =100 (1), 150 (2), 200 (3), 300 (4), 400 (5), the value of K is labeled (6). The dashed curve corresponds to S_s = 400, $dU = 20$ V. The dotted curve corresponds to S_s = 400, $dU = 50$ V. The calculations are performed for such parameters: $n_e = 2 \cdot 10^{11}$ m^{-3}, $\chi_n = 0.5$, $T_e = 2.8 \cdot 10^3$ K, $\beta = 1.3$, $\mathrm{V} = 7500$ m/s. As one may notice, the rate of asymptotic approach of D_K to a constant value in the model (1)–(3) is significantly lower compared to the model of single-species ions (4) with ion mass of m_{mod}. Thus, at $S_s \ge 300$ and $U_{iz} \approx$ 300 V, the value of D_K differs from the asymptotic value by more than 4 %.

Since high bias potentials are not convenient for practical use, we consider further $U_{iz} \le 200$ V. The calculation results revealed that under ionospheric conditions with such bias potentials, the ratio of the IPS's electrode areas S_s should be less than 200.

The results presented in Fig. 1 show that at $\chi_n = 0.5$ the limiting constant value of D_K within the model (1) – (3) is almost 1.5 times greater than the value of K calculated within the model of single-species ions (4). The numerical and analytical studies of the model (1) – (3) in the electron saturation region showed that when calculating K, the model ion mass m_{mod} should have a correction factor of

$2.1\chi_n(1-\chi_n)+1$. Further, we will designate with an asterisk such parameters calculated with this correction factor. The mass of the model ions and the value of K then writes as follows:

$$m^*_{\text{mod}} = [2.1\chi_n(1-\chi_n)+1]\cdot m_{\text{mod}}, \quad K^* = \frac{m^*_{\text{mod}}V^2}{2e} + \frac{kT_i}{2e} + \frac{\pi kT_e}{4e}.$$

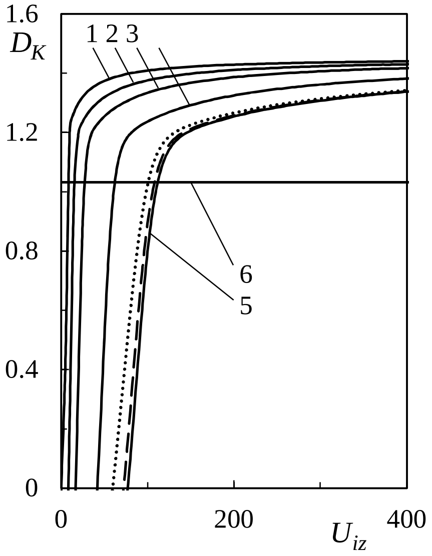

Fig. 1

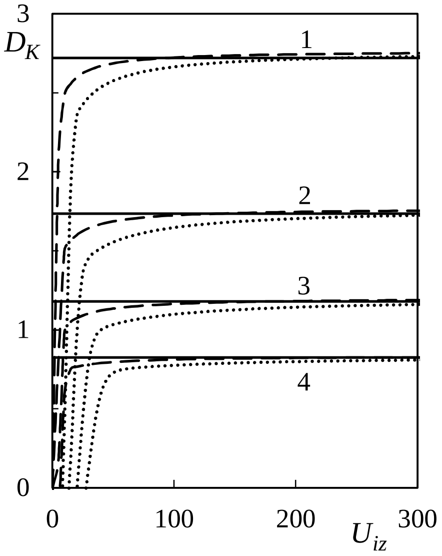

Fig. 2

Fig. 2 shows the dependence of D_K (in volts) on the bias potential U_{iz} (in volts) calculated by the model (1) – (3) for various S_s and χ_n at dU = 10 V in comparison with K^*. The groups of curves correspond to: χ_n = 0.2 (1), 0.4 (2), 0.6 (3), 0.8 (4). The solid curves represent K^*, dashed curves correspond to S_s = 100, and the dotted curves – S_s = 200. Analysis of the above results shows that at the electrode area ratio S_s = 100, the value of D_K approximates K^* quite well for U_{iz} within the range (100 – 200) V. Under the same conditions, for S_s = 200 the difference between D_K and K^* reaches ~5 %.

Based on the results of numerical modeling, a relationship is obtained between the temperature, bulk velocity of ions and electrons temperature of a supersonic plasma flow and measured probe currents in the electron saturation region within the framework of the plasma model with two ions species (1) – (3):

$$\frac{m^*_{\text{mod}}V^2}{2e} + \frac{kT_i}{2e} + \frac{\pi kT_e}{4e} \approx \frac{I_p^2(U_{iz})\cdot dU}{I_p^2(U_{iz}+dU) - I_p^2(U_{iz})} - U_{iz} + f(U_{iz}, S_s). \tag{7}$$

where $f(U_{iz}, S_s)$ is the correction to the relation (5). For the electrode areas ratio S_s from 100 to 200, the bias potentials U_{iz} from 100 V to 200 V, at $dU \geq 10$ V the correction term (in volts) is:

$$f\left(U_{iz}, S_s\right) = 0.032 \cdot \left[1.009 \cdot \left(0.626 + \left(\frac{S_s}{100} - 1 \right)^2 \right) \cdot \left(3.32 - \frac{U_{iz}}{100V} \right) - 1 \right].$$

Let’s consider the following quantities:

$$K_i^* = \frac{m_{\text{mod}}^* V^2}{2e} + \frac{kT_i}{2e}, \quad K_T^* = \frac{kT_i}{2e} + \frac{\pi k T_e}{4e},$$

$$D_K^* = \frac{I_p{}^2(U_{iz}) \cdot dU}{I_p{}^2(U_{iz} + dU) - I_p{}^2(U_{iz})} - U_{iz} + f(U_{iz}, S_s).$$

The previously introduced K^* along with K_i^*, K_T^* are complexes of kinetic parameters of charged plasma particles. Within the framework of the mathematical model with two ions species (1) – (3), the complex K^* characterizes the average kinetic energy of charged particles (in eV), K_i^* – the average kinetic energy of ions, and K_T^* – the average kinetic energy of thermal motion of charged particles (thermal energy). From (7) it follows that $K^* \approx D_K^*$, $K_i^* \approx D_K^* - \pi k T_e/(4e)$, $K_T^* = D_K^* - m_{\text{mod}}^* V^2/(2e)$. Thus, D_K^* is an experimentally determined quantity that estimates the average kinetic energy of charged plasma particles in the electron saturation region.

Note that D_K^* is always positive and for a stationary plasma flow within the framework of the considered model of two ions species plasma (1) – (3) in the electron saturation region it weakly depends on the bias potential U_{iz} and the electrode areas ratio S_s during measurement. The value of D_K^* is straightforwardly determined applying the standard method of processing the measured probe current I_p of the IPS with specified electrode areas ratio S_s at various bias potentials U_{iz} in the electron saturation region (6).

Methodological error of the calculation formula (7) within the framework of the mathematical model of two ions species plasma is estimated by calculating D_K^* using the current $I_p(U_{iz})$ which is the solution of problem (1) – (3), and comparing that obtained value of D_K^* with K^*.

Fig. 3 shows the dependence of the relative error $\overline{\varepsilon}_D = \left(D_K^* - K^*\right)/K^*$ of K^* determination by (7) on the bias potential U_{iz} (in volts) at $dU = 10$ V and $S_s = 100$ for various $\chi_n = 0$ (1), 0.2 (2), 0.5 (3), 0.7 (4), 0.8 (5), 0.9 (6), 1 (7).

Fig. 4 shows the dependence of $\overline{\varepsilon}_D = \left(D_K^* - K^*\right)/K^*$ on the electrode areas ratio S_s for various χ_n and U_{iz} at $dU = 10$ V. The groups of curves correspond to $\chi_n = 0.2$ (1), 0.5 (2), 0.8 (3). Solid curves corresponds to $U_{iz} = 100$ V, dashed curves – to $U_{iz} = 150$ V, dotted curves – to $U_{iz} = 200$ V.

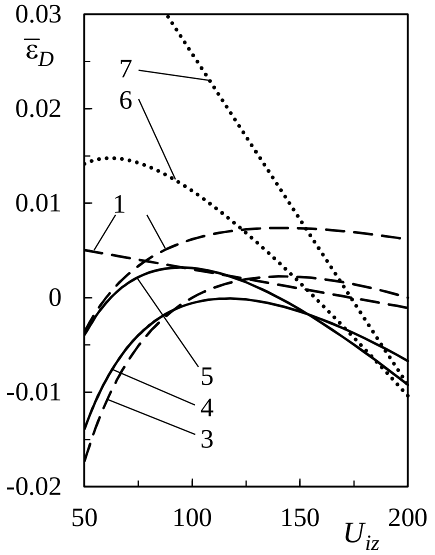

Fig. 3

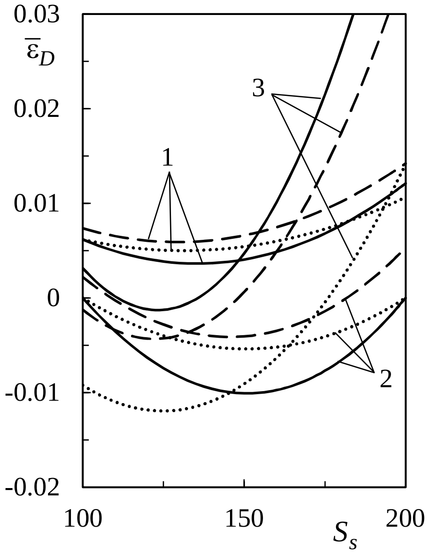

Fig. 4

Within the current collection model (1) – (3), the error $\bar{\varepsilon}_D$ shown in Figures 3 and 4 is a methodological error of calculation K^* using formula (7). The results presented in Figures 3, 4 reveal that for S_s from 100 to 160, U_{iz} from 60 V to 200 V and $\chi_n < 0.8$, the relative error $\bar{\varepsilon}_D$ does not exceed ~ 1 %.

Probe measurement errors. Let us consider the effect of errors in measuring currents and voltages by the IPS on the error in determining D_K^*. Let for the IPS with the electrode areas ratio S_s as a result of measuring the bias potential U_{iz} and the corresponding probe current $I_p(U_{iz})$, the approximate values are obtained, respectively:

$$\tilde{U}_{iz} = U_{iz}(1+\tilde{\varepsilon}_U),\ \tilde{I}_p(\tilde{U}_{iz}) = I_p(\tilde{U}_{iz})(1+\tilde{\varepsilon}_I), \tag{8}$$

where $\tilde{\varepsilon}_U$, $\tilde{\varepsilon}_I$ are random values on the intervals $[-\varepsilon_U, \varepsilon_U]$, $[-\varepsilon_I, \varepsilon_I]$ respectively; ε_U, ε_I are the maximum relative errors in measuring the corresponding quantities (ε_U, $\varepsilon_I > 0$).

Numerical analysis of the mathematical model of current collection in plasma with two ions species (1) – (3) revealed the following relations for the dimensional potentials and currents in the electron saturation region:

$$I'_p(U_{iz})/I_p(U_{iz}) \approx \frac{1}{2}\cdot\frac{1}{K^*+U_{iz}},\quad \frac{I_p(U_{iz})\cdot I'_p(U_{iz})}{I_p^2(U_{iz}+dU)-I_p^2(U_{iz})} \approx \frac{1}{2}\cdot\frac{1}{dU},$$

$$\frac{I_p^2(U_{iz}+dU)+I_p^2(U_{iz})}{I_p^2(U_{iz}+dU)-I_p^2(U_{iz})} \approx \frac{2K^*+(2U_{iz}+dU)}{dU}. \tag{9}$$

At $U_{iz} > 90$ V, the relative error of relations (9) does not exceed 0.01 %.

Substituting approximate values $\tilde{U}_{iz}$, $\tilde{I}_p\left(\tilde{U}_{iz}\right)$ into the expression for D_K^*, and neglecting the correction term $f\left(U_{iz}, S_s\right)$ and the second order small members, after the straighforward transformations taking into account (9) and the accepted notation, we obtain the following estimate:

$$\left|\frac{\tilde{D}_K^* - D_K^*}{D_K^*}\right| \leq \varepsilon_D \approx 4\left(\frac{U_{iz}}{K^*} + 1\right)\left(\frac{K^*}{dU} + \frac{U_{iz}}{dU} + 1\right) \cdot \varepsilon_I\,, \tag{10}$$

where $\tilde{D}_K^*$ is the value of D_K^* calculated from the measured currents and voltages (8), ε_D is the maximum relative error in determining D_K^*. The error ε_U is not present in the estimate (10), since for small measurement errors (8), the effect of the bias voltage error ε_U is the second order small.

In (10), the relative error ε_D does not depend explicitly on the electrode areas ratio S_s. However, the total error includes the methodological error of formula (7), that does depend on S_s, as one can see from Figures 3 and 4. The kinetic parameters of the plasma and the ion composition χ_n influence on ε_D in (10) through K^*.

It follows from (10) that the error ε_D monotonically decreases as the bias potential U_{iz} decreases and the potential increment dU increases. At $dU >> K^*$ an increase in the potential increment dU does not lead to any decent decrease in ε_D. Analysis of the results shows that for an adequate determination of the average kinetic energy of charged plasma particles, the maximum relative error in measuring the probe current and bias potential should not exceed ~1 %.

Kinetic parameters of plasma. The complexes K^*, K_i^* and K_T^* characterize the kinetic energy of charged particles, therefore they are determined by local plasma parameters and do not depend on probe measurements. As shown above, these complexes are estimated through D_K^*. Thus, we determine $\tilde{D}_K^*$ from the results of probe measurements and using (7), (10) we estimate the complex K^* of kinetic parameters of charged plasma particles:

$$\tilde{K}^* \approx \tilde{D}_K^*\,, \quad \left|\frac{\tilde{K}^* - K^*}{K^*}\right| \leq \varepsilon_K = \varepsilon_D\,,$$

where $\tilde{K}^*$ is the value of K^* calculated by the results of probe measurements, ε_K is the relative error in determining K^*, ε_D is estimated in (10).

Let the electron temperature $\tilde{T}_e$ be specified with the maximum relative error ε_{T_e}. Then the complex of kinetic parameters of ions K_i^* writes as follows:

$$\tilde{K}_i^* \approx \tilde{D}_K^* - \frac{\pi k \tilde{T}_e}{4e}\,, \quad \left|\frac{\tilde{K}_i^* - K_i^*}{K_i^*}\right| \leq \varepsilon_{K_i} \approx \frac{K^*}{K_i^*}\varepsilon_D + \frac{\pi k T_e}{4eK_i^*}\varepsilon_{T_e}\,,$$

where $\tilde{K}_i^*$ is the value of K_i^* calculated from the results of probe measurements, ε_{K_i} is the relative error in determining K_i^*.

Within the framework of the considered model of IPS's current collection, it is quite difficult to separate the contribution of ion's bulk and thermal motion energies to K_i^*. Usually, special experiments are carried out to determine either the temperature T_i or the mass velocity V of ions [1, 12].

If the temperature of the ions $\tilde{T}_i$ is given with the maximum relative error ε_{T_i}, then the mass velocity V is determined as follows:

$$\tilde{V} = \sqrt{2\left(\tilde{K}_i^* - k\tilde{T}_i/2\right)\Big/\tilde{m}_{\text{mod}}^*} \ ,$$

$$\left|\frac{\tilde{V} - V}{V}\right| \le \varepsilon_V \approx \frac{1}{2}\left(\frac{K_i^*}{m_{\text{mod}}^* V^2/2e}\varepsilon_{K_i} + \frac{kT_i/2e}{m_{\text{mod}}^* V^2/2e}\varepsilon_{T_i} + \varepsilon_m\right),$$

where $\tilde{V}$ is the value of V calculated from the results of probe measurements, ε_V is the relative error in determining V, ε_m is the relative error in determining m_{mod}^*. The error in determining the model ions mass m_{mod}^* holding the first order small members is estimated by:

$$\left|\frac{\tilde{m}_{\text{mod}}^* - m_{\text{mod}}^*}{m_{\text{mod}}^*}\right| \le \varepsilon_m \approx \left(\frac{3.5}{6}\sqrt{\frac{\mu_{\text{mod}}^*}{\mu_2}} + \frac{2.3}{3}\right)\sqrt{\frac{\mu_{\text{mod}}^*}{\mu_{\text{mod}}}}\varepsilon_\mu ,$$

where $\tilde{m}_{\text{mod}}^*$ is the value of m_{mod}^* calculated from the results of probe measurements when determining the plasma ion composition [9], , ε_m is the relative error in determining m_{mod}^*, ε_μ is the relative error in determining the parameter μ_{mod}, $\mu_{\text{mod}}^* = m_e \big/ m_{\text{mod}}^*$. The value of ε_μ is estimated in [9].

If the ion mass velocity $\tilde{V}$ is given with the maximum relative error ε_V, then the complex K_T^* of ions and electrons temperatures is determined as follows:

$$\tilde{K}_T^* = \tilde{D}_K^* - \frac{\tilde{m}_{\text{mod}}^* \tilde{V}^2}{2e}, \quad \left|\frac{\tilde{K}_T^* - K_T^*}{K_T^*}\right| \le \varepsilon_{K_T} \approx \frac{K^*}{K_T^*}\varepsilon_D + \frac{m_{\text{mod}}^* V^2}{2eK_T^*}\left(\varepsilon_m + 2\varepsilon_V\right),$$

where $\tilde{K}_T^*$ is the value of K_T^* calculated by the results of probe measurements, ε_{K_T} is the relative error in determining K_T^*.

If reliable estimates of the complex K_T^* and the electron temperature $\tilde{T}_e$ are given with maximum relative errors ε_{K_T} and ε_{T_e}, respectively, then the ion temperature T_i is estimated as follows:

$$\tilde{T}_i = \frac{2e}{k}\left(\tilde{K}_T^* - \frac{\pi k\tilde{T}_e}{4e}\right), \quad \left|\frac{\tilde{T}_i - T_i}{T_i}\right| \le \varepsilon_{T_i} \approx \frac{K_T^*}{kT_i/2e}\varepsilon_{K_T} + \frac{\pi}{2}\frac{T_e}{T_i}\varepsilon_{T_e} ,$$

where $\tilde{T}_i$ is the value of T_i calculated by the results of probe measurements, ε_{T_i} is the relative error in determining T_i.

The proposed calculation relationships make it possible to organize monitoring of the kinetic parameters of ionospheric plasma using probe systems on ultra-small satellites.

Conclusions. The possibility of determining the kinetic parameters of charged particles in plasma with two ions species based on separate current measurements using the insulated probe system in the electron saturation region is theoretically substantiated. Based on the previously obtained asymptotic solution for the electron saturation current in one-component plasma, a relationship is obtained between the measured probe current and the temperature and mass velocity of ions, the temperature of electrons. Numerical and analytical study of this relationship within the framework of a mathematical model of plasma with two ions species made it possible to find calculation formulas for determining complexes of kinetic parameters of charged particles based on separate current measurements using the insulated probe system in the electron saturation region.

The errors of the calculation formulas for determining the kinetic parameters of plasma with two ions species are estimated numerically and analytically for various parameters of the insulated probe system, the probe bias potential relative to the reference electrode potential, the accuracy of measuring probe currents and potentials. It is shown that for estimating the kinetic parameters of charged particles in a supersonic plasma flow, it is reasonable to choose the electrode areas ratio from 100 to 160, the bias potential from 100 V to 200 V and the potential increment of greater than 10 V.

The presented results can be used in the diagnostics of ionospheric plasma.

1. *Chung P. M., Talbot L., Touryan K. J.* Electric Probes in Stationary and Flowing Plasmas. Springer-Verlag, 1975. 150 p. https://doi.org/10.1007/978-3-642-65886-0
2. *Shuvalov V. A., Lazuchenkov D. N., Gorev N. B., Kochubei G. S.* Identification of seismic activity sources on the subsatellite track by ionospheric plasma disturbances detected with the Sich-2 onboard probes. Advances in Space Research. 2018. V. 61, No. 1. P. 355–366. https://doi.org/10.1016/j.asr.2017.08.001
3. *Akhoondzadeh, M., Parrot, M., and Saradjian, M. R.* Electron and ion density variations before strong earthquakes (M>6.0) using DEMETER and GPS data, Nat. Hazards Earth Syst. Sci. 2010. V. 10. P. 7–18. https://doi.org/10.5194/nhess-10-7-2010
4. *Jiang, S-B., Yeh, T-L., Liu, J-Y., Chao, C-K., Chang, L.C., Chen, L-W., Chou, C-J., Chi, Y-J., Chen, Y-L., Chiang, C-K.* New algorithms to estimate electron temperature and electron density with contaminated DC Langmuir probe onboard CubeSat. Advances in Space Research. 2020. V. 66, No. 1. P. 148–161. https://doi.org/10.1016/j.asr.2019.11.025
5. *Ranvier S., Lebreton J.-P.* Laboratory measurements of the performances of the Sweeping Langmuir Probe instrument aboard the PICASSO CubeSat. Geosci. Instrum. Method. Data Syst. 2023. V. 12, No. 1. P. 1–13. https://doi.org/10.5194/gi-12-1-2023
6. *Boyd R.* Langmuir Probes on Spacecraft. In: Plasma Diagnostics. W. Lochte-Holtgreven (Ed.). New York : AIP Press, 1995. P. 732–776.
7. IRI. Version: 2020. URL: https://ccmc.gsfc.nasa.gov/models/IRI~2020/
8. *Lazuchenkov D. N., Lazuchenkov N. M.* Mathematical modeling of probe measurements in a supersonic flow of a four-component collisionless plasma. Tech. Mech. 2020. No. 4. P. 97–108. https://doi.org/10.15407/itm2020.04.097
9. *Lazuchenkov D. N., Lazuchenkov N. M.* Mathematical modeling of determination the ionospheric plasma charged particles density by the electric current measurements using an insulated probe system. Tech. Mech. 2024. No. 2. P. 112–123. https://doi.org/10.15407/itm20241.02.112
10. *Lazuchenkov D. N., Lazuchenkov N. M.* Calculation of the ion current to a conducting cylinder in a supersonic flow of a collisionless plasma. Tech. Mech. 2022. No. 3. P. 91–98. https://doi.org/10.15407/itm2022.03.091
11. *Lazuchenkov D. N., Lazuchenkov N. M.* Determination of kinetic parameters of supersonic plasma flow of a gas-discharge source from current measurement by an insulated probe system. Tech. Mech. 2023. No. 4. P. 40–49. https://doi.org/10.15407/itm2023.04.040
12. Plasma Diagnostics. W. Lochte-Holtgreven (Ed.). New York : AIP Press, 1995. 945 p.

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