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ESTIMATION OF THE DEORBIT TIME OF SPACECRAFT AND SPACE DEBRIS FROM LEO

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Метою цієї статті є розробка процедури оцінки терміну зведення з низької майже кругової навколоземної орбіти космічних апаратів та об'єктів космічного сміття. Сформульовано у зручному для практичних розрахунків вигляді рівняння руху тіла по круговій орбіті при дії на нього атмосферного тертя та при наявності заданої активної сили. На етапі спостереження, за доступними онлайн даними про висоту орбіти та з використанням модельної густини атмосфери апроксимований балістичний коефіцієнт тіла, значення якого на практиці майже завжди невідомо. На етапі оцінки проведено розрахунок процесу деорбітінгу з використанням раніше знайденої апроксимації балістичного коефіцієнта. Визначено похибку в розрахунку терміну деорбітінгу в залежності від тривалості етапів спостереження та оцінки. Проведено аналіз точності розрахунку часу деорбітінгу для різних моделей густини атмосфери. Розроблена процедура дозволяє оцінити мінімально необхідне прискорення, яке забезпечує зведення з орбіти тіла протягом заданого часу. Працездатність процедури підтверджено розрахунками деорбітінгу космічного апарата, який перебував у некерованому польоті протягом 16 років. Термін деорбітингу оцінено з похибкою близько 1 % від фактичного терміну. Запропонована процедура може бути використана для оцінки терміну перебування на орбіті об'єктів космічного сміття, а також при плануванні активного деорбітінгу космічних апаратів наприкінці терміну їх експлуатації. Запропонована процедура може бути основою для майбутнього розвитку, враховуючи інші взаємодії між тілом на орбіті та космічним середовищем.

Ключові слова: *зниження орбіти, космічне сміття, оцінка часу сходу космічного апарата з орбіти, атмосферний опір, модель густини атмосфери.*

The goal of this article is to develop a procedure for estimating the deorbit time of used spacecraft and space debris from a nearly circular low-Earth orbit. The paper presents, in a convenient for practical calculations form, an equation of motion of a body along a circular orbit under the action of the atmospheric drag and a given active force. At the observation stage, available online data on the orbit altitude and a model atmospheric density allow one to approximate the ballistic coefficient of the body, which is nearly always unknown in practice. At the estimation stage, the deorbit process is calculated using the approximate ballistic coefficient found previously. The deorbit time calculation error is determined as a function of the duration of the observation and estimation stages for different atmospheric models. The proposed procedure allows one to estimate the minimum acceleration to deorbit the body in a given time. The procedure is validated by calculating the deorbit time of a spacecraft whose uncontrolled flight lasted 16 years: the error is about 10 per cent of the actual deorbit time. The proposed procedure may be used in estimating the life time of space debris objects and in planning the active deorbit of spacecraft at the end of their service life. The proposed procedure may be a basis for future development with account for other interactions between an orbiting body and the space environment.

Keywords: *orbital decay, space debris, spacecraft deorbit time estimation, atmospheric drag, atmospheric density model.*

Introduction. Currently, there is a rapid increase in the number of satellites in low Earth orbits (LEO) and very low Earth orbits, mainly due to commercial telecommunications spacecraft. LEO became the one of the most populated areas in space, both with active spacecraft and space debris objects (SDO) which is the remains of rockets and decommissioned spacecraft. To counteract the increase in the number of objects in orbit that pose a danger to active spacecraft, it is necessary to ensure deorbiting of decommissioned spacecraft within 25 years (international rule [1]), 5 years (USA rule [2]). The overwhelming majority of spacecraft, after the end of their service life, leave its orbit in an uncontrolled mode, and only about half of the objects subjected to the debris mitigation requirements do comply [3].

When designing a spacecraft, it is necessary to determine the costs of maintaining it in a given orbit and end-of-life disposal. In this regard, the problem of determining the deorbit time depending on the forces acting on it is relevant.

There is a well-developed tool OSCAR (Orbital Spacecraft Active Removal) within the DRAMA software package for the compliance analysis of a space mission with space debris mitigation standards [4]. OSCAR is capable to calculate deorbit fuel requirements for a given initial orbit and disposal scenario. Inputs are parameters of orbit, type of fuel, cross-sectional area (which often needs to be separately calculated) and mass of the satellite and the output is evolving orbital parameters over the whole deorbit phase calculated by solving 3D motion equations numerically.

The proposed procedure uses past data on the altitude of the body at the almost circular LEO as an input and calculates the future altitude using the energy balance equation, thus estimates the duration of passive or active deorbit. This procedure due to its comparatively high computational speed can be applied to optimization problems for which the deorbit time is a target function.

Problem formulation. The input data for the deorbiting problem is the dependence of the altitude h of a body's nearly circular orbit on time t over a certain interval $t_0 \le t \le t_1$. An object is considered deorbited if its orbital altitude decreases to 150 km. To determine the deorbiting time, we therefore need to determine the orbital altitude at subsequent times $t > t_1$ and find the time t_2 at which $h(t_2) = h_2 \le 150$ km.

We assume that the forces acting on the body vary little over a long period of time (compared to the rotation period which is about 1.5 hours at LEO), so that the orbit remains circular.

Solution to the problem. We consider the body rotating in a circular orbit and the force applied to it directed along the velocity vector with the magnitude that can be considered constant over some time Δt. We write the law of conservation of the total energy of the body's orbital motion over time Δt as

$$a_{12} = -\frac{\sqrt{\mu}}{\Delta t}\left(\frac{1}{\sqrt{r_2}} - \frac{1}{\sqrt{r_1}}\right). \tag{1}$$

Here, a_{12} is the force-induced acceleration along the direction of motion; μ is the Earth Gravitational parameter; r_1 is the radius of orbit from which the body is replaced to the orbit with radius r_2 during the time Δt.

Among the natural forces acting on a body, we consider only the force of aerodynamic drag, since it has a dissipative nature and makes the greatest contribution to the deorbiting of objects at altitudes of up to 1000 km [5]. According to [6], in an Earth Centered Inertial reference frame, the acceleration of aerodynamic drag equals

$$\vec{a}_d = -\frac{\rho}{2}\frac{C_D A}{m} v_{rel}^2 \frac{\vec{v}_{rel}}{\left|\vec{v}_{rel}\right|},$$

where ρ is a density, C_D is a drag coefficient, A is the cross-sectional area, m is the mass of the orbiting object, $\vec{v}_{rel}$ is the velocity vector of the body relative to the atmosphere.

In the deorbiting problem, the values of C_D, A, m are almost always undefined. All three are usually combined in the ballistic coefficient

$C^* = C_D A/m$, which characterizes the interaction of an object in orbit with the atmosphere.

When calculating the aerodynamic drag force to determine the velocity of a spacecraft relative to the atmosphere, it is usually assumed that the lower layers of the atmosphere rotate with the Earth. The angle between $\vec{v}$ and $\vec{v}_{rel}$ varies in orbit in the range of [0, γ], where $\gamma = \left| \arctan\left(\frac{T_S}{T_E} \sin\theta \Big/ \left(1 - \frac{T_S}{T_E} \cos\theta \right) \right) \right|$, T_S is orbital period, T_E is the period of the Earth's rotation around its axis, θ is an orbit inclination. For the conservative estimation of the deorbit time (we slightly underestimate drag) we calculate the relative velocity as the projection of $\vec{v}_{rel}$ to $\vec{v}$ as follows:

$$v_{rel} = v\left(1 - \frac{T_S}{T_E} \cos\theta \right)$$

and further consider that drag acceleration is acting along the direction of motion. Considering the above assumptions, the aerodynamic drag acceleration writes

$$\vec{a}_d = -a_d \frac{\vec{v}}{|\vec{v}|}, \quad a_d = \frac{\rho}{2} C^* v_{rel}^2 . \qquad (2)$$

In the case of active deorbiting, we assume that there is an additional active force that creates an acceleration a_f directed along the direction of motion

$$\vec{a}_f = -a_f \frac{\vec{v}}{|\vec{v}|} . \qquad (3)$$

The acceleration in (1) applied to the body, taking into account the adopted assumptions, writes

$$a_{12} = -a_d - a_f . \qquad (4)$$

There are physical and empirical atmospheric models which are being refined by the space and ground measurements [7], [8]. In this work, we use the following models in order to compare results:

Density model 1: Altitude profiles of the atmospheric density at the equatorial region [11]. Profiles are tabulated for low, moderate and high solar and geomagnetic activity. Interpolation by the three is performed using the actual solar activity index $F_{10.7}$ [10].

Density model 2: Altitude profiles but without a priori $F_{10.7}$ data. Constant value or approximation / forecast for $F_{10.7}$ is used to select profiles.

Density model 3: NRLMSISE-00 [9], [7].

The proposed procedure consists of two phases.

<u>*Observation phase*</u>. Let us denote the observation time χ from the moment t_0 to t_1. Based on the data on the body's orbital altitude over time $r(t)$, and atmospheric density model $\rho(r,t)$, we determine the values of $C^*(t)$ from (1)–(4). We will approximate ballistic coefficient $C^*(t)$, with the function

$$C^{**}(t,c_i)=c_0+c_1\cos\left(2\pi(t-t_0)/11+c_2\right), \tag{5}$$

where t, t_0 are measured in years. The form of function (5) is chosen to take into account the possible influence of the known 11-year cycle of solar activity. Having solved the problem of selecting parameters

$$\min_{c_i}\left\|C^{**}(t,c_i)-C^{*}(t)\right\|_2^2 \ , \ t_0\le t\le t_1,$$

using the least squares method, we obtain the values c_0, c_1, c_2.

<u>*Estimation phase*</u>. Let's denote the estimation time δ from t_1 to t_2. We calculate the body's orbital altitude at times $t_1<t\le t_2$ by (1) – (4), using approximation (5) for C^* in (2) and the very same atmospheric density model which is used during the observation phase. When determining the atmospheric parameters in the future, the solar index $F_{10.7}$ approximation by the recent data or long term forecast [12], [13] need to be used.

We define the error in calculating the deorbiting time ε as follows:

$$\varepsilon(\delta)=t^*-t_2 \ , \ \delta=t_2-t_1,$$

where t^* is the estimated time of reaching the orbital altitude at which the body is actually located at time t_2.

The introduced time values are shown schematically in Fig. 1, where the solid curve is the natural orbital altitude, and the dashed curve is the calculated altitude.

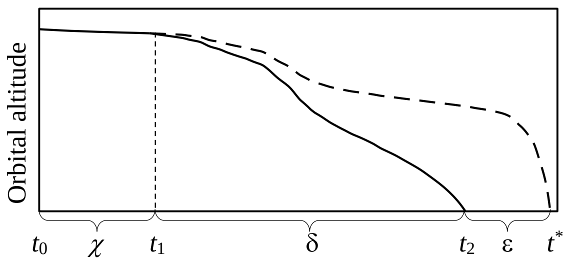


Fig. 1

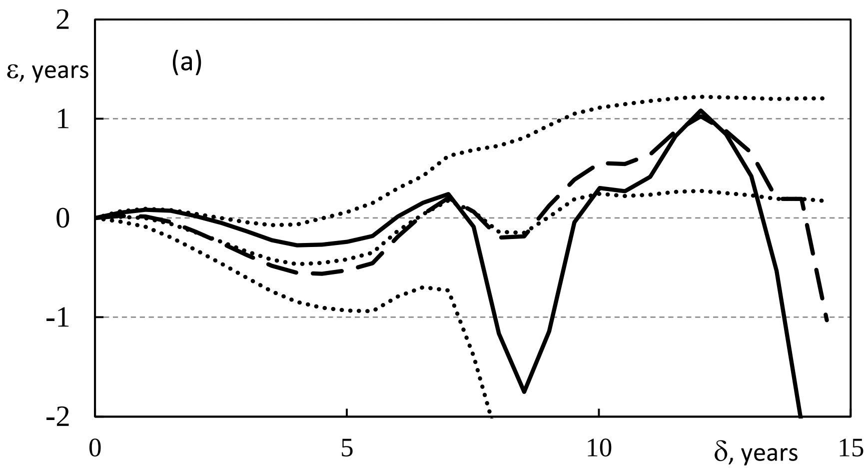
The acceleration a_f that must be applied to a body in the direction opposite to its velocity to ensure a predetermined deorbiting time can be found by solving

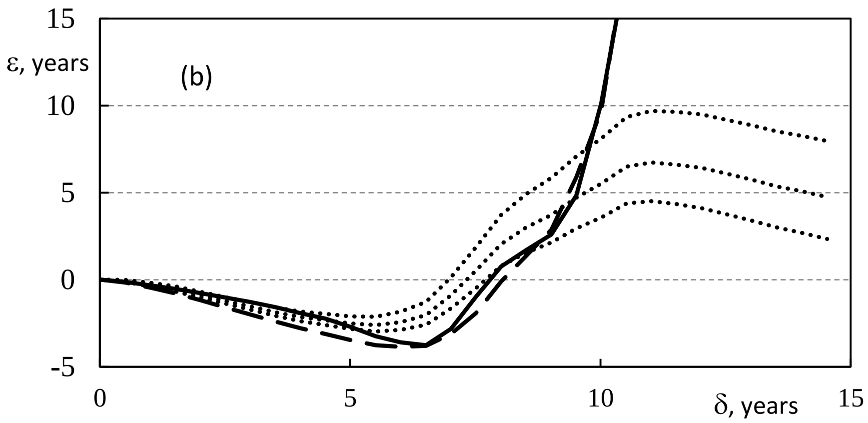
$$T\left(C^*,x\right)-T^*=0, \ x\in[0,a_{\max}]$$

approximately using the bisection method. Here, $T\left(C^*,x\right)$ is the calculated deorbit time of the body with ballistic coefficient C^* approximated by (5) with applied active acceleration of x, T^* is the required deorbit time, $a_{\max}$ is any large enough value, for example 10^{-5} m/s^2.

Validation of the proposed procedure is performed by applying it to the ERBS satellite (Norad number 000015354) [14]. The dependence of the orbital altitude on time is used [10]. It is known that starting from 2006 the motion of the satellite is uncontrolled, a_f = 0 [14].

Fig. 2 presents the error in estimating the deorbiting time ε (in years) depending on the estimation time δ (in years) for t_0 = 2006, t_2 = 2022 for different density models. The observation phase duration varies with δ as χ =16–δ years. Here, the dashed curves correspond to $c_1 = c_2 = 0$ in approximation (5). Plot 2(a) calculated using density model 1, dotted curves correspond to C^*=0.012, 0.014, 0.016 m^2/kg (from upper to lower); Plot 2(b) – density model 2 with $F_{10.7}$ =95, dotted curves correspond to given a priori constant ballistic coefficients C^*=0.02, 0.03, 0.04 m^2/kg (from upper to lower); Plot 2(c) – density model 2 with $F_{10.7}$ =98.34–33.79·cos(2π(t–2006)/11–1.3256), time t – in years, dotted curves correspond to C^* =0.012, 0.014, 0.016 (from upper to lower); Plot 2(d) – density model 3 (NRLMSISE-00), dotted curves correspond to C^*=0.010, 0.012, 0.014 m^2/kg (from upper to lower).





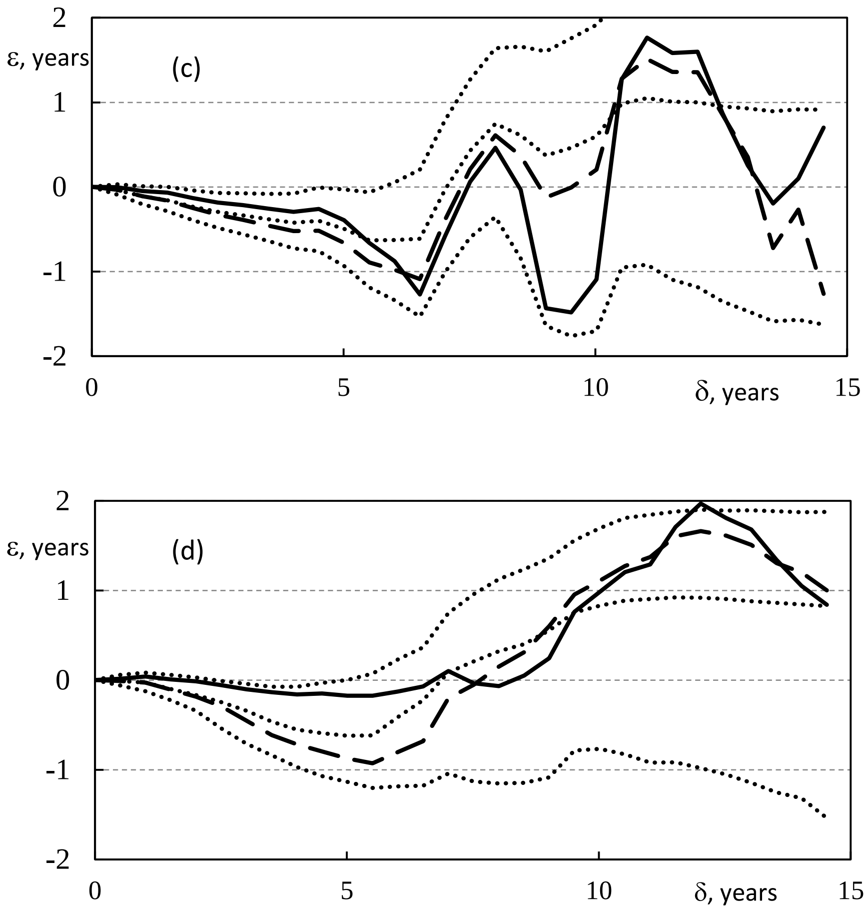


Fig. 2

From the above plots we can see that for long duration of observation phase χ >9 years (δ<7 years) approximation (5) improves the accuracy of deorbit time estimation compared to the case of $c_1 = c_2 = 0$ in (5). Calculations for a priori given constant C^* (dotted curves) show that satellite's C^* isn't constant for all considered density models (we would expect $\varepsilon / \delta \approx$ Const otherwise).

Fig. 3 shows the relative error ε / δ in estimating deorbiting time calculated using density model 1 (solid curve); model 2 for $F_{10.7}$ = 98.34–33.79·cos(2π(*t*–2006)/11–1.3256), time *t* – in years (dashed curve); model 3 (dotted curve). Here, t_0 = 2006, t_2 = 2022.

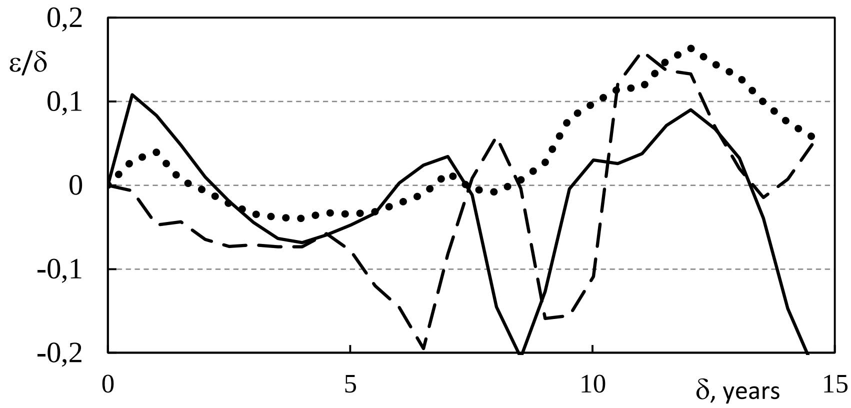


Fig. 3

We notice that using the model 3 (NRLMSISE-00) is preferable over all other considered and the error is less than about 10% of the total deorbit duration.

Fig. 4 shows the density ρ (in kg/m^3) calculated by the considered models at the real ERBS altitude. Thick curve represents density model 1; dotted curve – density model 2 for $F_{10.7}$ = 95; dashed curve – density model 2 for $F_{10.7}$ = 98.34 – 33.79·cos(2π(t–2006)/11–1.3256), time t – in years; thin curve – density model 3 (NRLMSISE-00).

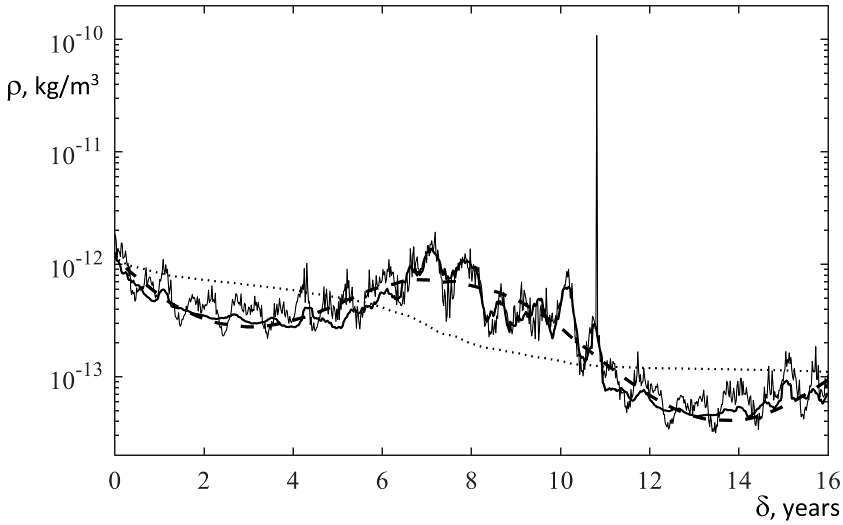


Fig. 4

Fig. 5 shows the error in estimating the deorbiting time ε (in years) depending on the estimation time δ (in years) for $t_2 = 2022$ and various fixed χ values. The solid curve corresponds to $\chi = 9$ years, the dashed line to $\chi = 6$ years, and the dotted line to $\chi = 3$ years. The atmospheric density model 3 (NRLMSISE-00) is used.

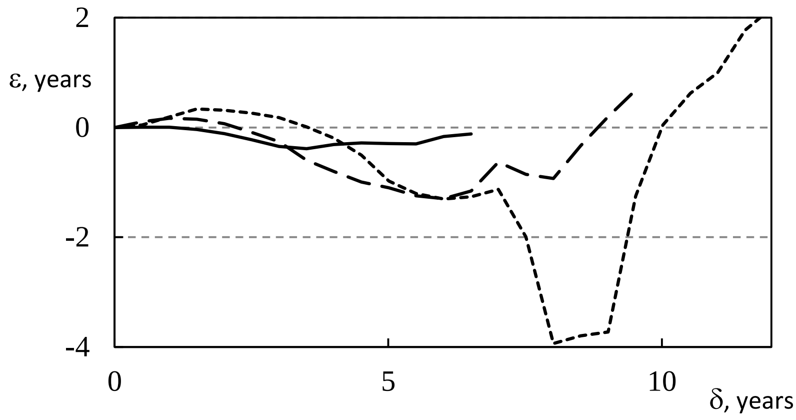


Fig. 5

Fig. 6 shows the orbital altitude profiles h (in km) calculated since 2010 for the ERBS spacecraft at various constant values of a_f. The solid curves correspond to the initial boundaries (0, 10^{-6}) m/s^2 of the a_f range, thin dashed curves are intermediate calculations using the bisection method, and the thick dashed curve is the result of determining $a_f = 2.35 \cdot 10^{-7}$ m/s^2 to ensure spacecraft deorbiting within 5 years.

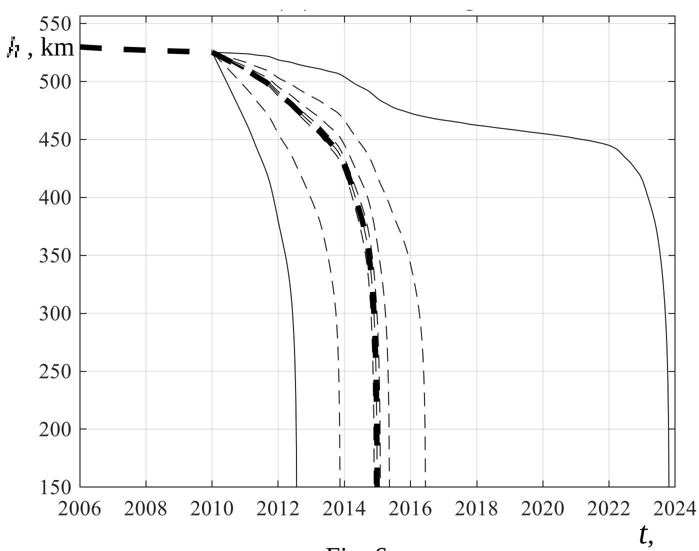


Fig. 6

Conclusions. A procedure for estimating the deorbiting time of a body from a circular LEO is proposed. At the observation phase, using the given orbital parameters the ballistic coefficient which characterizes the influence of atmospheric drag on the body motion is approximated. At the estimation phase, the body's orbital altitude is calculated using the obtained ballistic coefficient. The error in determining the deorbiting time depending on the duration of the observation and estimation phases for several atmospheric density models is studied.

A comparison is made of the results of calculating the passive deorbiting of the ERBS spacecraft at an inclination of approximately 60° over 16 years using various atmospheric density models. The deorbiting time is estimated with an error of about 10% of the actual deorbiting duration using NRLMSISE-00 and about 15% using tabulated altitude density profiles. Density calculations using the NRLMSISE-00 model ensure more precise estimation of deorbit instant, but are the most resource-intensive. Tabulated density model with actual data on solar activity performs worse but can be used to save calculation time. Simple approximation of $F_{10.7}$ using only recent observation data and tabulated density profiles makes it possible to calculate the deorbit time with 15-20% error. Constant value of $F_{10.7}$ isn't applicable at al due to strong influence of the solar activity on atmosphere density. Proposed approximation of the ballistic coefficient for long duration of the observation phase provided more precise estimation of the deorbit time compared to the constant value.

Using the ERBS satellite as an example, the proposed procedure is shown to estimate the required additional acceleration for active deorbiting over a given time.

The proposed procedure can be a basis for future development by including other interactions between orbiting body and space environment.

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