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**D. N. LAZUCHENKOV, N. M. LAZUCHENKOV**

# MATHEMATICAL MODELING OF ELECTRON DENSITY DETERMINATION USING A STATIONARY CYLINDRICAL LANGMUIR PROBE UNDER IONOSPHERIC CONDITIONS

*The Institute of Technical Mechanics of National Academy of Sciences of Ukraine and State Space Agency of Ukraine, 15 Leshko-Popelya St., Dnipro 49005, Ukraine; e-mail: lazuch.dn@gmail.com*

Метою статті є теоретичне обґрунтування застосування класичної формули теорії одиночного циліндричного зонда для визначення концентрації електронів за окремими вимірюваннями струмів плаваючої зондової системи в умовах іоносфери. Моделювання зондових вимірювань в іоносфері виконано на прикладі циліндричних зонда і корпусу надмалого супутника, що розташовані поперечно в надзвуковому беззіштовхувальному потоці плазми. Іоносферна плазма вважається максвелівською, складається з електронів та однозарядних атомарних іонів кисню та водню. Розроблено математичну модель збирання електричних струмів плаваючою зондовою системою "зонд – плазма – корпус супутника". Модель побудована на основі класичних співвідношень для електронного та іонного струмів на тонкий циліндр, що поперечно обтікається. Для моделювання збирання корпусом супутника іонів водню отримано апроксимацію результатів числових розрахунків іонного струму на циліндр за двовимірною моделлю Власова–Пуассона. Знайдено значення потенціалів зсуву зонда, за яких в умовах іоносфери електронна область вольтамперної характеристики плаваючої зондової системи найбільш близька до вольтамперної характеристики одиночного зонда. Виконано моделювання визначення концентрації електронів за класичною розрахунковою формулою теорії одиночного зонда при вимірюваннях зондових струмів на низьковольтній ділянці електронної області вольтамперної характеристики зондової системи. Отримано граничні оцінки методичної похибки визначення концентрації електронів в умовах іоносфери в рамках моделі розглянутої зондової системи. Досліджено вплив похибок виміру зондового струму на визначення концентрації електронів за розрахунковою формулою теорії одиночного зонда. Отримані результати можуть бути використані при підготовці та інтерпретації експериментів із діагностики іоносферної плазми з використанням надмалих супутників.

***Ключові слова:*** *іоносферна плазма, іони атомарного кисню та водню, надмалі космічні апарати, плаваюча зондова система, одиночний циліндричний зонд, математична модель збирання струму, достовірність визначення концентрації електронів.*

The goal of this article is to theoretically substantiate the applicability of the classical formula of the single cylindrical probe theory to determining the electron density based on measurements of the currents of a floating probe system in ionospheric conditions. Probe measurements in the ionosphere are modeled using a cylindrical probe and the body of a very small satellite placed transversely in the incident supersonic flow of a collisionless plasma. The ionospheric plasma is considered to be Maxwellian and consists of electrons and singly charged atomic ions of oxygen and hydrogen. A mathematical model of current collection by the floating probe system “probe – plasma – satellite body” was developed based on classical relationships for the electron and ion currents to a thin cylinder placed transversally in the incident flow. To model the collection of the hydrogen ion current by the satellite body, the results of numerical calculations of the ion current to the cylinder using the two-dimensional Vlasov-Poisson model were approximated. The probe bias potential was determined such that, under ionospheric conditions, the electron region of the floating probe system's current-voltage characteristic is closest to that of a single probe. The determination of the electron density using the classical calculation formula of the single-probe theory was simulated for probe current measurements in the low voltage portion of the electron region of the current-voltage characteristic of the floating probe system. The limiting methodological error in determining the electron density under ionospheric conditions within the framework of the probe system model considered was estimated. The effect of probe current measurement errors on the determination of the electron density using the calculation formula of the single-probe theory was studied. The obtained results may be used in the preparation and interpretation of ionospheric plasma diagnostics experiments using ultra-small satellites.

***Key words****: ionospheric plasma, atomic oxygen and hydrogen ions, ultra-small spacecraft, floating probe system, single cylindrical probe, mathematical model of current collection, reliability of electron density determination.*

**Introduction**. Stationary cylindrical Langmuir probes are traditionally used for ionospheric plasma diagnostics on spacecraft [1 – 4] due to the simplicity of the measuring equipment design, well-developed theory, and acceptable accuracy of measurements of local plasma parameters (electron density and temperature) in the vicinity of the spacecraft.

Exposed part of the spacecraft body is typically used as the reference electrode for the probe. The probe measurement system onboard the spacecraft is floating – the reference electrode (spacecraft) potential acquires such value that the total current on the spacecraft equals zero [5]. It is known [5, 6] that to utilize the classical relationships of a single cylindrical probe in the electron saturation regime (at high bias potentials of the probe relative to the spacecraft), the area of the reference electrode must be 3-4 orders of magnitude greater than the area of the probe. In the ionosphere, researchers often conduct measurements in the low voltage side of the electron part of the current-voltage characteristic (CVC), at bias potentials of about 10 V, where the requirement for the ratio of the reference and measuring electrodes areas is less strong [1 – 4].

Currently, the miniaturization of electronics and power supplies offers promising prospects for the use of stationary cylindrical Langmuir probes for organizing global ionospheric monitoring using a large number of inexpensive nano satellites. However, trying to increase the signal level (probe current) conflicts with the limitations of the theory of a single stationary Langmuir probe [5]. Therefore, quantitative estimating the applicability of the single-probe theory onboard nano satellites is relevant.

In [7 – 8], ionospheric plasma diagnostics using a floating (isolated from the spacecraft body) probe system is theoretically substantiated, and a mathematical model for current collection is developed for an arbitrary ratio of the reference and measuring electrodes areas. Calculation formulas for determining the plasma's charged particle density are obtained for the electron saturation region, i.e., for sufficiently high probe bias potentials. This article provides a theoretical substantiation for the applicability of the relations of the single cylindrical probe theory to determining the electron density in the low voltage side of the electron region of the CVC of a floating probe system. The influence of the electrodes areas ratio and electric current measurement errors on the reliability of electron density determination is estimated.

**Problem formulation**. We model probe measurements onboard a very small spacecraft under the following assumptions. The spacecraft body is a fairly long circular cylinder with a base radius of $r_{cp}$ with conductive side surface and end surfaces insulated from the plasma. The probe is a long circular cylinder with a significantly smaller base radius $r_p$, $r_p = r_{cp}$. From here on, the subscript "*cp*" refers to the spacecraft (counter probe), and the subscript "*p*" refers to the probe.

Ionospheric plasma is a weakly ionized gas mixture whose charged particles are electrons and singly charged atomic ions of oxygen $O^+$ and hydrogen $H^+$. For the scale of the probe measurement problem, the interaction of charged and neutral components of the gas mixture can be neglected. The unperturbed plasma is considered to be Maxwellian, quasi-neutral, and nonisothermal. The ion temperatures are identical, $T_{H^+} = T_{O^+} = T_i$, and the degree of nonisothermality is characterized by the parameter $\beta = T_e / T_i$, where $T_e$ is the electron temperature. The ion composition of the plasma is characterized by the parameter $\chi_n = n_{H^+} \Big/ \left( n_{H^+} + n_{O^+} \right) \equiv n_{H^+} / n_e$, where $n_{H^+}$, $n_{O^+}$ are the densities of ions $H^+$ and $O^+$, respectively, $n_e$ is the electron density.

During probe measurements, the spacecraft body is used as a reference electrode. We consider that the axes of symmetry of the spacecraft body and probe are perpendicular to the orbital velocity $\mathrm{V}$; the electrostatic and gas dynamic interactions between the probe and the spacecraft in the plasma are weak; the side surfaces completely absorb the charge of incident particles (electrons are absorbed, ions are neutralized), and there is no emission current from the surfaces; the flow around the probe and the spacecraft is free-molecular; the effect of the magnetic field on the probe current is negligible; the presence of different species of ions in the plasma does not significantly alter the self-consistent electric field in the vicinity of the cylinder.

The main geometric parameter of the considered floating probe system is the current-collecting surface areas ratio $S_s = S_{cp}/S_p$ , where $S_{cp}$, is the area of the reference electrode, $S_p$ is the area of the probe ($S_p << S_{cp}$). The theory of a single cylindrical probe corresponds to the case $S_s \to \infty$. The purpose of this article is to evaluate the applicability of the relationships of the theory of a single cylindrical probe for determining the electron density $n_e$ by measuring probe currents in the low voltage side of the electron region of the CVC of a floating probe system.

**Mathematical model of current collection**. The theory of supersonic free-molecular transverse plasma flow around a long conducting cylinder is familiar and studied quite well. The main parameters determining the flow regime are: the ion velocity ratio $S_i = V/u_i$ , the ratio of the cylinder base radius $r_c$ to the Debye length $\xi = r_c/\lambda_d$ , and the dimensionless potential of the surface relative to the undisturbed plasma potential $\varphi_c = eU_c/kT_e$. Here, $V$ is the flow velocity, $u_i = \sqrt{2kT_i/m_i}$ is the thermal velocity of ions of mass $m_i$, $U_c$ is the dimensional potential of the surface, $k$ is the Boltzmann constant, and $e$ is the unit charge.

During orbital motion in the ionosphere, the velocity ratio is $S_i \geq 4$ for oxygen ions and $S_i \leq 2$ for hydrogen ions.

In the ionosphere, the Debye length for electrons is $\lambda_d \approx 5$ mm. Consequently, the characteristic size of a cylindrical probe of $r_p \approx 0.5$ mm corresponds to $\xi_p = r_p/\lambda_d \approx 0.1 << 1$, and the characteristic size of a spacecraft (nano satellite) of $r_{cp} \approx 5$ cm corresponds to $\xi_{cp} = r_{cp}/\lambda_d \approx 10$.

Thus, ion flow around the probe occurs at $S_i \geq 4$, $\xi <<1$ for $O^+$, and $S_i \leq 2$, $\xi <<1$ for $H^+$; ion flow around the reference electrode (spacecraft body) occurs at $S_i \geq 4$, $\xi \approx 10$ for $O^+$, and $S_i \leq 2$, $\xi \approx 10$ for $H^+$.

There is a classical Langmuir asymptotic relation $\sqrt{-\varphi_c}$ for the ion current in supersonic flow around a thin cylinder ($\xi << 1$) with ion-attracting surface potential $\varphi_c$ [6, 9]. The applicability of the Langmuir asymptotic relation for the flow parameters of $S_i \geq 1$, $\xi \leq 1$ and $S_i > 3$, $\xi \leq 10$ for the body potential of $-50 < \varphi_c < 0$ is substantiated based on the results of numerical modeling [10].

A mathematical model of current collection by a cylindrical electrode in a supersonic flow of collisionless plasma with two-species ions is developed in [7 – 8]. The model is developed on the basis of asymptotic formulas for electron and ion currents on a thin cylinder [9, 11] taking into account the results of works [12 – 13, 14]. In dimensionless quantities, the total current on the cylinder with the potential $\varphi$

relative to the undisturbed plasma potential, is estimated by the relations [8] (the electron current on the cylinder is positive):

$$\bar{I}_c(\varphi) = \bar{I}_e(\varphi) - 4\chi_n\sqrt{\mu_2/\beta}\cdot\bar{I}_{H^+}(\varphi) - (1-\chi_n)\sqrt{\mu_2/\beta}\cdot\bar{I}_{O^+}(\varphi),\ S_i > 4, \quad (1)$$

$$\bar{I}_e(\varphi) = \begin{cases} 2/\sqrt{\pi}\cdot\sqrt{\pi/4+\varphi}, & \varphi > 0 \\ \exp(\varphi), & \varphi \le 0 \end{cases}, \quad (2)$$

$$\bar{I}_{O^+}(\varphi) = \begin{cases} \sqrt{2/\pi}\exp\left(-\beta\varphi + S_i^2\right), & \beta\varphi \ge S_i^2 \\ 2/\sqrt{\pi}\cdot\sqrt{1/2 + S_i^2 - \beta\varphi}, & \beta\varphi < S_i^2 \end{cases}, \quad (3)$$

$$\bar{I}_{H^+}(\varphi) = \begin{cases} \sqrt{2/\pi}\exp\left(-\beta\varphi + S_i^2/16\right), & \beta\varphi \ge S_i^2/16 \\ 2/\sqrt{\pi}\cdot\sqrt{1/2 + S_i^2/16 - \beta\varphi}, & \beta\varphi < S_i^2/16 \end{cases}, \quad (4)$$

where $\bar{I}_c$, $\bar{I}_e$ are the total and electron currents on the cylinder, respectively, which are normalized by the thermal electron current; $\bar{I}_{O^+}$, $\bar{I}_{H^+}$ are ion currents on the cylinder, respectively, which are normalized by the thermal currents of ions of corresponding species; $\varphi = eU/kT_e$ is the dimensionless electric potential ($U$ is the dimensional potential); $\mu_2 = m_e/m_{O^+}$ is the mass ratio of charged particles; $S_i = V/u_{O^+}$ is the velocity ratio for $O^+$ ions . The thermal current of particles of species $\alpha$ is $I_{\alpha,0} = j_\alpha S_c$, where $j_\alpha = en_\alpha u_\alpha/2\sqrt{\pi}$ is the density of the thermal current, $u_\alpha = \sqrt{2kT_\alpha/m_\alpha}$ is the thermal velocity, $T_\alpha$ and $m_\alpha$ are the temperature and mass of the particles, $S_c$ is the area of the current-collecting surface of the cylinder. From here on, the index $\alpha = i$ refers the value to the ions, the index $\alpha = O^+$ to atomic oxygen ions, the index $\alpha = H^+$ to atomic hydrogen ions, and the index $\alpha = e$ to electrons.

Relations (1)–(4) fairly well approximate the current collection by the cylinder at $\xi = 1$, $S_i \ge 1$ and, within the framework of the adopted assumptions, can be used to model the currents on the stationary cylindrical probe. These same relations are applicable to currents on cylinder at $\xi \le 10$, $S_i > 3$. Therefore, to model the electron and $O^+$ ions currents to the reference electrode (spacecraft body) at potentials $\varphi > -50$, relations (2) and (3) are used.

To model the collection of $H^+$ ions current by the spacecraft body, we use the results of numerical calculations of ion currents to the cylinder using the two-dimensional Vlasov-Poisson model [10, 14] at $1 \le \xi \le 10$, $1 \le S_i \le 3$ and potentials $-50 < \varphi < 0$. Based on the results of the numerical calculations, the ion current to the cylinder is approximated as follows:

$$\bar{I}_{H^+}(\varphi) = \begin{cases} \dfrac{2}{\sqrt{\pi}} \exp\left(-\beta\varphi + \log\left(\sqrt{1/2 + S_i^2/16}\right)\right), & \beta\varphi \ge 0 \\ \dfrac{2}{\sqrt{\pi}} \left[\left(1/2 + S_i^2/16\right)^{\frac{1}{2\kappa}} - \beta\varphi\right]^{\kappa}, & \beta\varphi < 0 \end{cases}, \tag{5}$$

where

$$\kappa = 0.163\left[\frac{(\xi^2+96)(\xi^2-1)}{(\xi^2+71)(\xi^2+33)} + 0.38\right]\frac{(\xi^2+7)(S_i^2-1)}{(\xi^2+11)\,S_i^2 + 6(\xi^2+1)} + 0.315\frac{\xi^2+49}{\xi^2+33.2}.$$

Fig. 1 shows the results of calculating the dimensionless ion current $\bar{I}_i$ as a function of the dimensionless potential of the cylinder $\varphi$ at velocity ratios of $S_i$ =1 (Fig. 1, a)), $S_i$ =2 (Fig. 1, b)), $S_i$ =3 (Fig. 1, c)) for different $\xi$. Curves (approximations) and markers (calculations, see Fig. 1, c) for markers designation) correspond to $\xi$ = 1 (1), $\xi$ =1 (2, 6), $\xi$ =3 (3, 7), $\xi$ =5 (4, 8), $\xi$ =10 (5, 9, 10). Markers 6, 8, 9 represent calculations [14]; 7, 10 – calculations [10].

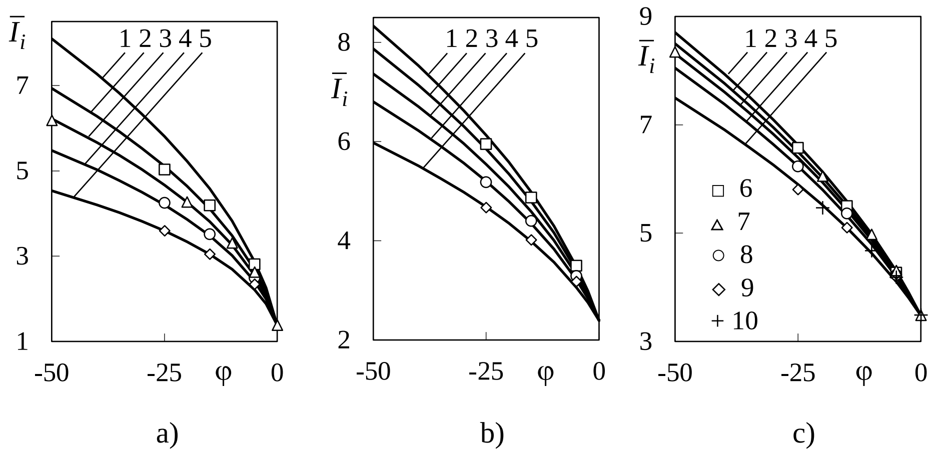


Fig. 1

As one can see at Fig. 1, relation (5) approximates the results of numerical calculations of ion current on the cylinder with satisfactory accuracy.

Thus, in a supersonic plasma flow with two ions species, the total current on the probe and the spacecraft body is determined by (1) through the electron current (2), the oxygen ion current (3), and the hydrogen ion current (4) for the probe and (5) for the spacecraft body.

**Direct problem of probe measurements**. A mathematical model for collecting currents by a floating probe system with cylindrical electrodes is developed in [7, 8]. In the ionosphere, the floating probe system always has such equilibrium potential, at which the total current of charged particles through all collecting surfaces of the electrodes equals zero [5]. The potential of the spacecraft body $U_{cp}$ relative to the undisturbed plasma potential is almost always negative. The probe potential relative to the undisturbed plasma is $U_p = U_{iz} + U_{cp}$, where $U_{iz}$ is the probe potential relative to the reference electrode (bias potential). Probe measurement is the recording the current $I_p$ in the "probe – plasma – reference electrode" circuit while modulating the bias potential $U_{iz}$.

In dimensionless variables, calculating the CVC of the probe $\bar{I}_p(\varphi_{iz})$, taking into account relations (1) – (5), reduces to a system of nonlinear equations [8]

$$\bar{I}_p(\varphi_{iz}) = \bar{I}_c(\varphi_{iz} + \varphi_{cp}), \qquad (6)$$

$$S_s \cdot \bar{I}_c(\varphi_{cp}) + \bar{I}_c(\varphi_{iz} + \varphi_{cp}) = 0, \qquad (7)$$

where $\varphi_{cp}$ is the equilibrium potential of the reference electrode relative to the undisturbed plasma potential, which corresponds to the bias potential $\varphi_{iz}$.

Since approximation (1)–(5) of the current $\bar{I}_c$ on the cylinder in a supersonic plasma flow is a continuous piecewise analytic function of the potential $\varphi$ and parameters $\chi_n$, $\mu_2$, $\beta$, $S_i$, the solution to the nonlinear equation (7) of the current balance of the floating probe system relative to the equilibrium potential $\varphi_{cp}$ exists and is unique for all values of the bias potential $\varphi_{iz}$ and parameter $S_s$. The solution to equation (7) can be found by an iterative method [7].

Thus, relations (6), (7), (1)–(5) determine the electrical and gas-dynamic interaction (the dimensionless VAC $\bar{I}_p(\varphi_{iz})$) in the floating "probe–plasma–reference electrode" system through the dimensionless parameters $\chi_n$, $\mu_2$, $\beta$, $S_i$, $S_s$ and the bias potential $\varphi_{iz}$.

The dimensionless parameters $\beta$, $S_i$, $S_s$, $\varphi_{iz}$ are determined through the parameters of the undisturbed plasma, probe, and reference electrode: $T_e$, $T_i$, $V$, $S_p$, $S_{cp}$, $U_{iz}$. The dimensional CVC of a floating probe system writes $I_p(U_{iz}) = j_e S_p \bar{I}_p(\varphi_{iz})$, where $j_e = e\sqrt{k/(2\pi m_e)}\, n_e \sqrt{T_e}$ is the density of the thermal electron current to the probe.

In [7, 8], the influence of electrodes areas ratio $S_s$ and plasma ion composition $\chi_n$ on the CVC of a probe system insulated from the spacecraft body is studied at sufficiently high bias potentials $\varphi_{iz}$. It is shown that in the electron saturation regime the influence of $\chi_n$ and $S_s$ on the collected probe current is the greatest.

Fig. 2 illustrates the influence of the bias potential $\varphi_{iz}$ and parameters $\chi_n$, $S_s$ on the probe current $\bar{I}_p$ of a floating probe system in the low voltage side of the electron region of the CVC. Calculations are performed for $S_i = 5$, $\mu = 3.4 \cdot 10^{-5}$ and $\beta = 1.3$. Fig. 2, a) is the dependence of $\bar{I}_p$ on $\varphi_{iz}$ for $\chi_n$=0.1 (solid curves), $\chi_n$=0.3 (dashed curves) for $S_s$= 100 (1), 150 (2), 200 (3), 250 (4), 2000 (5). Fig. 2, b) is the dependence of $\bar{I}_p$ on $\chi_n$ for $\varphi_{iz}$=40, $S_s$= 2000 (1), 300 (2), 250 (3), 200 (4), 150 (5), 100 (6) and for $\varphi_{iz}$=15, $S_s$= 2000 (7), 300 (8), 250 (9), 200 (10), 150 (11), 100 (12).

It is clearly seen that the influence of $\chi_n$ and $S_s$ on the probe current increases as the bias potential $\varphi_{iz}$ increases. Increasing the electrode areas ratio $S_s$ makes the

CVC $\bar{I}_p(\varphi_{iz})$ closer to the CVC of a single probe (for which $S_s \to \infty$). The CVC of the floating probe system approaches to the CVC of a single probe in the low voltage side of the electron region, at $\varphi_{iz}$ <40.

Let's model numerically the probe measurements for such base values of parameters $n_e = 2 \cdot 10^{11}$ m$^{-3}$, $T_e = 2.8 \cdot 10^3$ K, $\beta = 1.3$, $V = 7.5 \cdot 10^3$ km/s that correspond to flow conditions in the ionosphere at an altitude of about 700 km [15]. A bias potential of $\varphi_{iz} \approx 40$ corresponds to a dimensional potential of $U_{iz} \approx 10$ V and a probe current density of about 0.018 A/m$^2$.

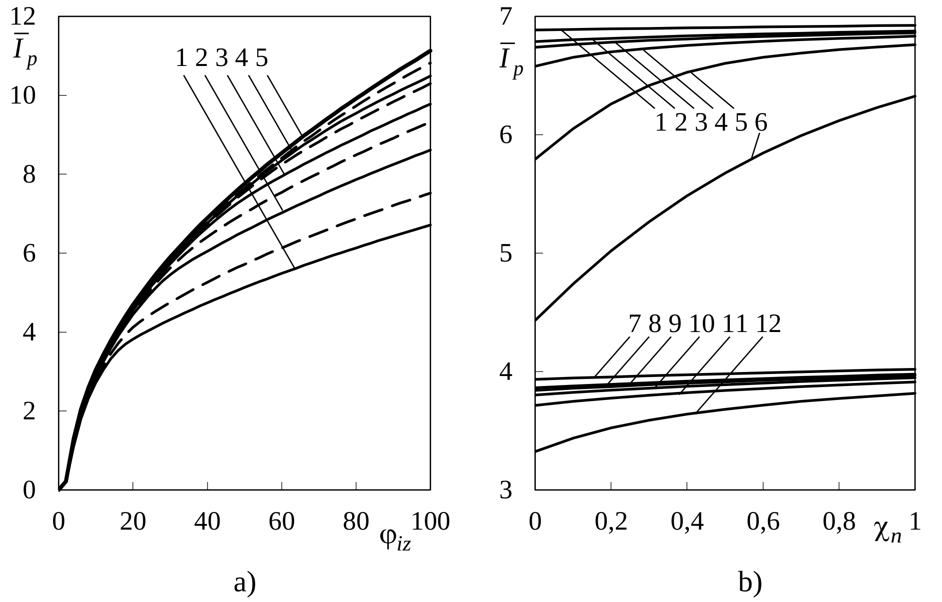


Fig. 2

**Inverse problem of probe measurements**. We model ionospheric measurements by a floating probe system using the mathematical model of current collection (6), (7), (1) – (5). Let's consider the problem of determining the electron density (parameter $n_e$ of the mathematical model) by the results of probe current measurements $I_p(U_{iz})$ in the low voltage side of the electron region of the CVC of the floating probe system. We use the calculation formula of the theory of a single stationary cylindrical probe [6] to determine $n_e$:

$$n_{e,0} = \frac{\pi}{eS_z}\sqrt{\frac{m_e}{2e}}\sqrt{\frac{dI_p^2(U_{iz})}{dU_{iz}}}. \qquad (8)$$

Due to the linearity of $I_p^2(U_{iz})$ in the electron region of the CVC, (8) writes as

$$n_{e,0} = \frac{\pi}{eS_z}\sqrt{\frac{m_e}{2e}}\sqrt{\frac{I_p^2(U_{iz}+dU) - I_p^2(U_{iz})}{dU}}. \qquad (9)$$

Here, $I_p(U_{iz})$ is the probe current, corresponding to the bias potential $U_{iz}$ within the framework of the single probe theory, $dU > 0$ is the bias potential increment.

**Methodological error** of the procedure for applying calculation formula (9) to determine the density $n_e$ within the framework of the mathematical model of current collection is estimated by calculating the value $\bar{n}_e$ by (9) using the currents found by solving (6), (7), (1) – (5) and comparing the result with $n_e$. In this case, the relative error $\bar{\varepsilon}_{n_e} = \left(\bar{n}_e - n_e\right)/n_e$ corresponds to the specific measurement conditions (bias potentials $U_{iz}$ during probe current $I_p\left(U_{iz}\right)$ measurements ) and the method of processing the results (the method of numerical differentiation of $I_p^2\left(U_{iz}\right)$ ).

The dependence of the relative methodological error $\bar{\varepsilon}_n$ on $S_s$ is presented at Fig. 3. Curves at Fig. 3, a) represent various $\chi_n$ = 0 (curve 1), 0.1 (2), 0.3 (3), 0.5 (4), 0.7 (5), 0.9 (6); Fig. 3, b) – various $V_i$ = 7000 (curve 1), 7500 (2), 8000 (3); Fig. 3, c) – various $T_e$ = 1500 (curve 1), 2000 (2), 2500 (3), 3000 (4), 3500 (5). The values of the parameters $\chi_n$, $T_e$, $V$ cover the ranges of their variation during the ionospheric measurements. The curves at plot 3, d) are the maximal values of the dependence $\bar{\varepsilon}_{n_e}$ on $S_s$ for the considered ranges of $\chi_n$, $V$, $T_e$ variations at various bias potentials $U_{iz}$ = 5V (curve 1), 10V (2), 15V (3), 20V (4).

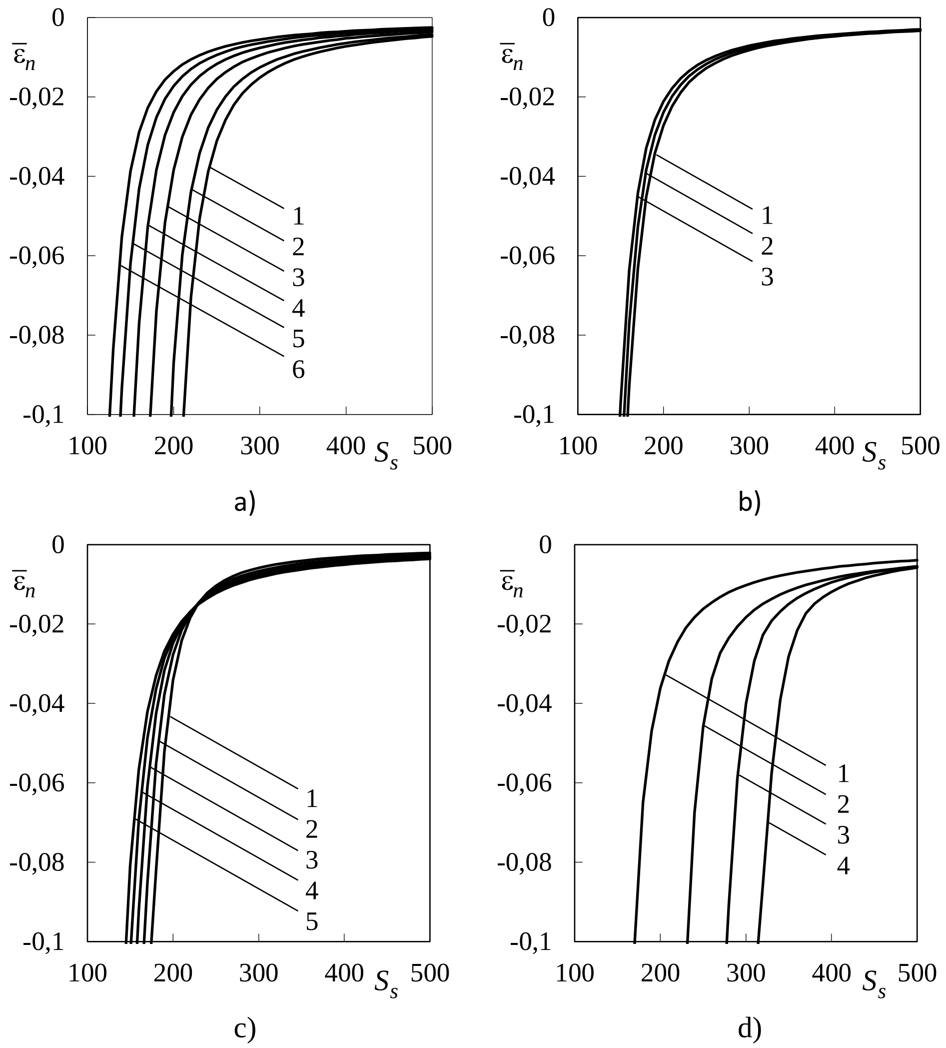


Fig. 3

Analysis of the results presented in Fig. 3 shows that with an increase in the electrodes areas ratio $S_s$ and a decrease in the bias potential $U_{iz}$ in the low voltage side of the electron region of the CVC, the methodological error $\bar{\varepsilon}_n$ decreases for all considered $\chi_n$, $T_e$, $V$. The effect of variation of parameters $\chi_n$, $V$ on the error in determining the density is monotonic for $S_s$>100, a variation in the electron temperature $T_e$, as it can be seen at Plot 3, c), leads to an intersection of the curves at $S_s \approx 220$.

Variation in the flow velocity $V$ does not significantly affect $\bar{\varepsilon}_n$. The influence of $\chi_n$ on $\bar{\varepsilon}_n$ is the strongest, and the maximum methodological error is observed at $\chi_n = 0$ (Plot 3, a)). The results presented at Plot 3, d) are the upper estimate of the methodological error $\bar{\varepsilon}_n$ for the procedure for determining the electron density in ionospheric conditions using the calculation formula (9) within the framework of the current collection model (6), (7), (1) – (5).

From the results of modeling it follows that at a bias potential of $U_{iz} \approx 10$ V, the required electrodes areas ratio is $S_s > 350$ to ensure a methodological error of $\bar{\varepsilon}_n \le$ 1%, $S_s \ge 300$ for $\bar{\varepsilon}_n \le$ 2%, $S_s \ge 250$ for $\bar{\varepsilon}_n \le$ 5%. The methodical error in determining the electron density $\bar{\varepsilon}_n <$ 4% at $U_{iz} \approx 5$ V and $S_s \ge 200$.

**Probe measurements error**. The structure of formula (9) is similar to calculation formulas for determining the electron density in a dissociated diatomic gas flow and plasma with single-species ions [6, 8]. For the electron density $\tilde{n}_e$ calculated from measured (obtained experimentally) probe currents $\tilde{I}_p(\tilde{U}_{iz})$, the following estimate satisfies:

$$\left|\left(\tilde{n}_e - n_e\right)/n_e\right| \le \varepsilon_n \approx \left(1 + 2U_{iz}/dU\right)\varepsilon_I . \tag{10}$$

Here, $\varepsilon_n$ is the maximum relative error of the determination $n_e$ using (9), $\varepsilon_I$ is the maximum relative error of the probe current measurement, $\left|\left(\tilde{I}_p - I_p\right)/I_p\right| \le \varepsilon_I$.

We notice that the relative error $\varepsilon_n$ doesn't depend explicitly on the geometric parameter $S_s$. However, the total error, including the methodological error of formula (9), as shown in Fig 3, depends on the electrodes areas ratio $S_s$. For the given $\varepsilon_n$ (accuracy of determining the electron density) we use (10) and the results presented at Plot 3, d) to select $S_s$, $U_{iz}$, $dU$ to estimate $\varepsilon_I$ (the required measurement accuracy).

It follows from (10) that increasing the potential increment $dU$ leads to a decrease in $\varepsilon_n$, which is typical for a numerical differentiation problem. As the bias potential $U_{iz}$ decreases, the error $\varepsilon_n$ decreases monotonically. However, in this case probe measurements occur in the transition region of the CVC, where high-precision measurement is difficult to conduct due to strong plasma noise.

**Conclusions**. The applicability in ionospheric conditions of classical relations of the single cylindrical probe theory for determining the electron density from individual current measurements in the low voltage side of the electron region of the CVC of a floating probe system is theoretically substantiated. A mathematical model is developed that determines the electrical and gas-dynamic interaction (CVC) in the "floating probe–plasma–spacecraft body" system through the parameters of the plasma, the probe system, and the probe bias potential. Calculations confirm that in the low voltage side of the electron region of the CVC, a decrease in the probe bias potential and an increase in the ratio of the spacecraft body and probe areas leads to a convergence of the CVC of the floating probe system and the CVC of a single probe.

A modeling of the determination of the electron density using the classical calculation formula of the single probe theory by measuring probe currents in the low voltage side of the electron region of the CVC of a floating probe system is performed. Upper estimates of the methodological error in determining the electron density under ionospheric conditions are obtained using the floating probe system model. It is shown that, for bias potentials not greater than 10 V and a spacecraft-to-probe areas ratio greater than 300, the methodological error does not exceed 2 %.

Presented numerical and analytical estimates of the electron density error enable the selection of the floating probe system's geometric parameters, probe bias potentials, and the assessment of the required measurement accuracy when planning and conducting ionospheric plasma diagnostic experiments.

The obtained results can be used in the preparation and interpretation of ionospheric plasma diagnostic experiments onboard ultra-small spacecraft.


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