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FEATURES OF THE MATHEMATICAL SIMULATION OF DYNAMIC PROCESSES IN THE RECONFIGURABLE PROPELLANT FEED HYDRAULIC SYSTEM OF LIQUID-PROPELLANT ROCKET ENGINES

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Гідравлічні трубопровідні системи в рідинних ракетних двигунах (РРД) є численними та різноманітними. У РРД з допалюванням генераторного газу відомі гідравлічні системи змінної структури, у яких під час запуску двигуна змінюється напрямок потоків компонентів палива. Для надійного запуску РРД необхідно забезпечити плавний перехід від живлення пусковим пальним до живлення основним пальним. Метою цієї роботи є розробка підходу до математичного моделювання динамічних процесів у розгалуженій гідравлічній системі живлення зі змінною структурою. Розроблено методичний підхід до математичного моделювання динамічних процесів у розгалуженій гідравлічній системі зі змінною структурою. Він передбачає визначення частотних характеристик кількох конфігурацій гідравлічної системи, які реалізуються на різних етапах її роботи, як системи з розподіленими параметрами. Надалі складається математична модель із зосередженими параметрами, що містить рівняння руху та нерозривності рідини у зосереджених параметрах. Зосереджені податливості розміщуються у розрахунковій гідравлічній схемі зазвичай у місцях розгалуження трубопроводів. Їхню кількість і значення задають таким чином, щоб частотні характеристики систем з розподіленими та зосередженими параметрами кожної конфігурації гідравлічної системи змінної структури узгоджувалися з заданою точністю. Для демонстрації запропонованого підходу розглянуто тестову гідравлічну систему змінної структури. Виділено дві конфігурації гідравлічної системи: від пускового бачка до газогенератора та від виходу з насоса до газогенератора, для яких визначено частотні характеристики для систем з розподіленими параметрами. Побудовано математичну модель динамічних процесів у аналізованій гідравлічній системі із зосередженими параметрами. Визначено значення зосереджених податливостей у вузлах, що дозволяють задовільно узгодити частотні характеристики з розподіленими та зосередженими параметрами. Встановлено, що значення зосереджених податливостей практично не змінюються за різних конфігурацій гідравлічної системи, граничних умов та співвідношень компонентів палива в газогенераторі.

Ключові слова: *рідинний ракетний двигун, система живлення змінної структури, розгалужений гідравлічний тракт, математичне моделювання, імпедансний метод, частотна характеристика*.

Hydraulic pipeline systems in liquid-propellant rocket engines (LPREs) are numerous and diverse. In staged-combustion LPREs, use is made of reconfigurable hydraulic systems, in which the propellant component flow directions change during an engine start-up. A reliable engine start-up requires a smooth transition from the start-up propellant supply to the main propellant supply. The objective of this study is to develop an approach to the mathematical simulation of dynamic processes in a branched reconfigurable hydraulic feed system. A methodological framework for modeling dynamic processes in such systems is proposed. It involves determining the frequency responses of several configurations of a hydraulic system realized at different stages of its operation as a distributed-parameter system. The next step is the construction of a lumped-parameter mathematical model with fluid motion and continuity equations in lumped parameters. Lumped compliances are typically introduced in the hydraulic network model at pipeline junctions. Their number and values are selected so that the frequency responses of the distributed- and lumped-parameter models for each hydraulic system configuration may be in agreement within a prescribed accuracy. To demonstrate the proposed approach, a test reconfigurable hydraulic system is considered. Two configurations of the hydraulic system are set off: from the start-up tank to the gas generator and from the pump outlet to the gas generator. For both configurations, frequency responses of the corresponding distributed-parameter systems are determined. A lumped-parameter mathematical model of dynamic processes in the hydraulic system under analysis is developed. The values of the lumped compliances at the network nodes are identified such that the frequency responses of the distributed- and lumped-parameter models are in satisfactory agreement. It is shown that the values of the lumped compliances remain practically unchanged for different hydraulic system configurations, boundary conditions, or propellant mixture ratios in the gas generator.

Keywords: *liquid-propellant rocket engine, reconfigurable feed system, branched hydraulic circuit, mathematical simulation, impedance method, frequency response.*

Introduction. Pipeline systems are widely used in industry, transportation, construction, and municipal utilities [1]. Particularly stringent reliability requirements are imposed when transporting liquids and gases in the chemical and petroleum industries, nuclear power engineering [2], and shipbuilding [3]. The most severe operating conditions, associated with structural vibrations of pipelines, occur in aviation and, especially, rocket technology [4, 5]. In liquid-propellant rocket engines (LPREs), over a relatively short time interval, the pipeline systems may first be exposed to cryogenic propellant components during filling and subsequently to high temperatures and intense heat fluxes from the regions where propellant combustion occurs, namely the gas generators and combustion chambers.

Hydraulic and pneumatic pipeline systems in LPREs are numerous and diverse. Mathematical modeling of dynamic processes in these systems is subject to the same stringent requirements as the modeling of individual LPRE units and assemblies. At present, mathematical simulation of LPRE operating processes shows a tendency to focus on individual engine components [6, 7] and to incorporate an extensive range of phenomena and effects observed at different stages of LPRE operation [8–10]. This enables effective in-service diagnostics of rocket engines [11], investigation of LPRE dynamic characteristics [12], decision-making on LPRE reuse [13], and development of LPRE control loops [14].

A well-known LPRE configuration with gas-generator afterburning and non-hypergolic propellant components employs a starting fuel for ignition [15 – 17]. This starting fuel is contained in a dedicated ampoule; after the ampoule is opened, the fuel is supplied to the ignition cavities of the gas generator and the combustion chamber, where it ignites spontaneously upon contact with the oxidizer. After LPRE start and transition to the main fuel, the hydraulic path that includes the starting-fuel ampoule is effectively shut off due to its high hydraulic resistance. In steady-state operation, the engine is supplied with the main fuel through the primary hydraulic line.

The described ignition scheme and the transition from starting fuel to the main fuel are based on the use of a reconfigurable propellant feed hydraulic system. For reliable LPRE start-up, it is necessary to ensure a smooth transition from starting-fuel supply to main-fuel supply. This transition changes the propellant mixture ratio in the gas generator and can significantly affect the stability of turbopump operating parameters during start-up.

The objective of this study is to develop an approach for mathematical modeling of dynamic processes in a branched reconfigurable propellant feed hydraulic system during its operation.

1. Mathematical Modeling of Dynamic Processes in Branched Pipelines. Pipeline systems in LPREs are often long and highly branched. The flow in these systems is predominantly turbulent, and wave processes may be significant. For most of these hydraulic systems, except for special cases, the characteristic flow velocity is much lower than the speed of sound; therefore, the convective terms in the unsteady momentum and continuity equations can be neglected [18]. On this basis, mathematical modeling of dynamic processes in LPRE pipeline systems should be formulated by solving the partial differential equations governing unsteady flow and continuity [18]:

$$\begin{cases} \dfrac{\partial p}{\partial z} + \dfrac{1}{A} \cdot \dfrac{\partial \dot{m}}{\partial t} + \dfrac{k}{A} \cdot \dot{m} = 0, \\ \dfrac{\partial \dot{m}}{\partial z} + \dfrac{A}{c^2} \cdot \dfrac{\partial p}{\partial t} = 0, \end{cases} \tag{1}$$

where p and – are the fluid pressure and mass flow rate; t – time; z – axial coordinate along the pipeline; A $\dot{m}$ – the pipeline cross-sectional area; k – the equivalent linear friction coefficient per unit pipeline length $k = \dfrac{2 \cdot \Delta\bar{p} \cdot A}{l \cdot \bar{\dot{m}}}$; $\Delta\bar{p}$ – the hydraulic pressure loss; l – the length of the pipeline section; $\bar{\dot{m}}$ –the steady-state mass flow rate; c – the speed of sound in the fluid flowing through a pipeline with elastic walls.

To solve system (1), the method of characteristics is often used [5, 19]. A less common approach is the impedance method [18], despite its successful application to a wide range of problems related to LPRE hydraulic system dynamics. In particular, the impedance method has recently been used to model the start-up of various propulsion systems [20], to simulate transient processes in a hydraulic system containing lumped gas cavities and undissolved gas in the liquid [20], and to analyze coupled longitudinal oscillations of the pipeline structure and the liquid.

For branched pipeline systems, the impedance method is more appropriate because it enables compact, universal models to be developed both for the initial LPRE start-up period during filling of the hydraulic lines with propellant components and for simulating dynamic processes during engine run-up to steady-state regime and under steady-state conditions. When the impedance method is applied, the solution of system (1) for a pipeline segment between nodes m and n can be represented as a passive two-port (four-terminal) network:

$$\begin{cases} \delta p_n = b_{mm} \cdot \delta p_m + b_{mn} \cdot \delta \dot{m}_m, \\ \delta \dot{m}_n = b_{nm} \cdot \delta p_m + b_{nn} \cdot \delta \dot{m}_m, \end{cases}$$

where δp_m and δp_n – are the pressure deviations at the inlet and outlet of the two-port network from their steady-state values; $\delta \dot{m}_n$ and $\delta \dot{m}_m$ – are the mass-flow-rate deviations at the inlet and outlet from their steady-state values of the two-port network; b_{mm}, b_{mn}, b_{nm} and b_{nn} – are the elements of the transfer matrix of the pipeline segment, which for a distributed-parameter system are defined as follows:

$$b_{mm} = ch(\gamma \cdot l), \quad b_{mn} = -Z_B \cdot sh(\gamma \cdot l), \quad b_{nm} = -\frac{sh(\gamma \cdot l)}{Z_B}, \; b_{nn} = ch(\gamma \cdot l),$$

where γ – complex wave propagation “constant” per unit pipeline length $\gamma = \sqrt{Z^* Y^*}$; Z_B – characteristic (wave) impedance of the pipeline, $Z_B = \sqrt{\dfrac{Z^*}{Y^*}}$; Z^* – hydraulic series impedance per unit pipeline length, $Z^* = \dfrac{1}{A}(j\omega + k)$; Y^* – shunt admittance (conductance) per unit pipeline length, $Y^* = \dfrac{A \cdot j\omega}{c^2}$; j – imaginary unit; ω – angular frequency.

Using the passive two-port network representation, the input impedance $Z_m(j\omega)$ and the gain (transfer) factor $W(j\omega)$ of the considered pipeline segment can be determined when the output impedance is known $Z_n(j\omega)$.

$$Z_m(j\omega) = \frac{\delta p_m(j\omega)}{\delta \dot{m}_m(j\omega)} = \frac{b_{mn} - b_{nn} \cdot Z_n(j\omega)}{b_{nm} \cdot Z_n(j\omega) - b_{mm}},$$

$$W(j\omega) = \frac{\delta p_n(j\omega)}{\delta p_m(j\omega)} = b_{mm} + b_{mn} \frac{1}{Z_1(j\omega)}.$$

The dynamic gain of the entire hydraulic system composed of pipeline segments con-nected in series is determined as the product of the gain factors of the individual segments. At branching points where three or more segments are connected, the frequency characteristics $Z(j\omega)$ and are obtained from the conditions of mass-flow balance for the incoming and outgoing flows, and from equality of pressures in all connected branches [18].

With boundary conditions specified at the ends of the pipeline system, the impedance-method approach presented enables the solution of system (1) for a pipeline network of virtually any complexity. The solution yields the natural (resonant) frequencies of fluid oscillations and the results of a stability analysis for the distributed-parameter pipeline system. In contrast to the method of characteristics, the impedance method also provides insight into the frequency range of the dynamic processes involved.

For a reconfigurable hydraulic system, the impedance-based approach presented requires refinement. It is proposed to consider several configurations of such a system that occur at different stages of its operation. The transition from one configuration to another during start-up simulation is accomplished either by the operation of the check valves or, in a forced manner, by programmatically closing some valves and opening others [21].

2. Lumped-Parameter Mathematical Model. To simulate transient processes in the hydraulic system during LPRE start-up, a lumped-parameter model of the hydraulic system dynamics is commonly used [4, 20]. Its construction relies on solutions of the differential equation system (1) obtained for the hydraulic line treated as a distributed-parameter system. In general, the proposed approach to mathematical modeling of dynamic processes in a branched reconfigurable propellant feed hydraulic system comprises several stages.

At the first stage, the frequency characteristics of several configurations of the reconfigurable hydraulic system that occur at different stages of its operation are determined by treating the system as a distributed-parameter system. Next, a lumped-parameter mathematical model is formulated, including the lumped-parameter forms of the fluid momentum and continuity equations.

$$\begin{cases} \delta p_n = \delta p_m + \left(R_m + j\omega J_m\right) \cdot \delta \dot{m}_m, \\ \delta \dot{m}_n = -j\omega C_m \, \delta p_m + \delta \dot{m}_m, \end{cases}$$

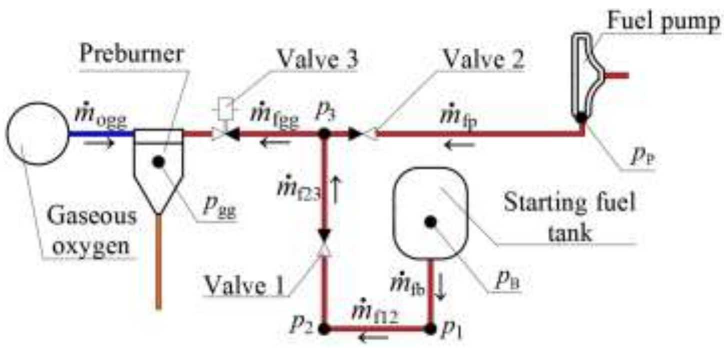
where R_m, J_m – are the coefficients of hydraulic (resistive) and inertial resistance of the pipeline segment; C_m – is the lumped compliance of the pipeline segment [22–24].

The coefficients J_m are determined from the geometric characteristics of the hydraulic lines, whereas the coefficients R_m are obtained from engine balance calculations or from the corresponding pressure-drop relations for hydraulic losses [25]. Lumped compliances are typically placed in the computational hydraulic scheme at pipeline branching nodes or at locations associated with long pipeline sections. The values of the compliance coefficients C_m are selected so that, for each configuration of the reconfigurable hydraulic system, the frequency characteristics of the distributed-parameter and lumped-parameter models match within a prescribed accuracy over a specified frequency range.

The lumped-parameter mathematical model is then used directly to simulate transient processes in the reconfigurable hydraulic system.

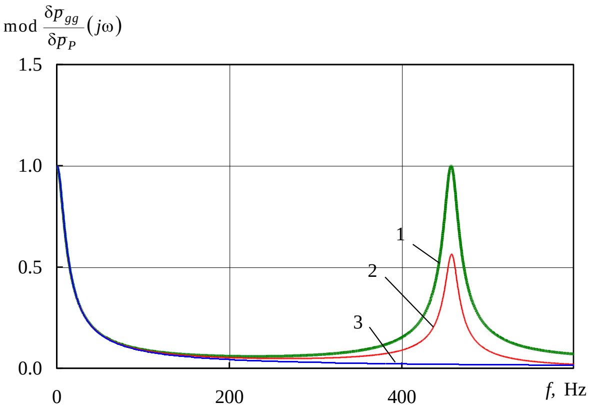
Compared with the method of characteristics [19], determining transient processes using the approach proposed in this work includes one additional intermediate stage, namely, the evaluation and matching of the frequency characteristics of the hydraulic system represented by distributed-parameter and lumped-parameter models. However, this stage is not redundant, because it enables a compact mathematical model to be constructed that describes the dynamic processes over a guaranteed frequency range, which is required for transient process simulation in a reconfigurable LPRE propellant feed system.

3. Example of a Reconfigurable Propellant Feed Hydraulic System. To demonstrate the proposed approach, we consider the simplest reconfigurable hydraulic system, namely the fuel feed system of the gas generator of a test restartable staged-combustion LPRE. This hydraulic system includes (see Fig. 1) three hydraulic lines: from the starting tank (p_B) to the branching node (p_3), from the pump outlet (p_p) to the branching node, and from the branching node to the gas generator (p_{gg}). The first and second lines contain check valves (labeled valve 1 and valve 2 in the scheme). The third line includes a solenoid valve, labeled valve 3, which is intended to isolate the fuel cavity from oxidizer vapors and purge gases.

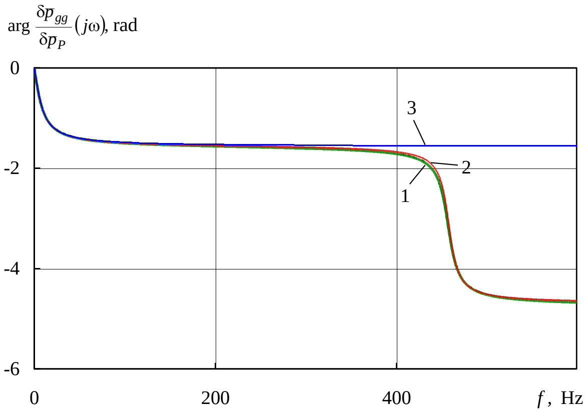


$\dot{m}_{ogg}$ – oxidizer flow rate to the gas generator;
$\dot{m}_{fgg}$, $\dot{m}_{fp}$, $\dot{m}_{fb}$, $\dot{m}_{f23}$, $\dot{m}_{f12}$ – fuel flow rates in the corresponding sections; p_1, p_2, p_3 – pressures at the nodes;
p_{gg} – pressure in the gas generator; p_B – pressure in the starting tank; p_p – pressure at the fuel pump outlet

Fig. 1 – Computational scheme of the feed system of the test LPRE



a



b

1 – distributed-parameter model; 2 – lumped-parameter model; 3 – lumped-parameter model with $C_j = 0$

Fig. 2 – Magnitude (a) and phase (b) of the gain factor

$$\frac{\delta \bar{p}_{\text{gg}}}{\delta \bar{p}_{\text{P}}}(j\,\omega)$$

The reconfigurable hydraulic system operates as follows. Upon a command from the control system, valve 3 is opened. At this initial time, the pump shaft is not yet spinning up, and the pressure at the fuel pump outlet p_p is much lower than the pressure in the starting tank p_B. Therefore, the fuel flows from the starting tank through the branching node (p_3) to the gas generator. Check valve 2 prevents fuel from entering the pump line. After ignition of the propellant components in the gas generator and sufficient spin-up of the pump shaft, the pressure at the fuel pump outlet p_p becomes higher than the starting-tank pressure p_B. Check valve 1 closes, the fuel flow from the starting tank stops, and the gas generator is supplied with fuel from the pump line.
Thus, during operation of the reconfigurable hydraulic system, the configuration of the hydraulic path supplying fuel to the gas generator changes. Initially, fuel flows from the starting tank (p_B) through the branching node (p_3) to the gas generator (p_{gg}). Subsequently, fuel is supplied from the pump (p_p) through the branching node (p_3) to the gas generator (p_{gg}). Therefore, mathe-matical modeling of dynamic processes in the reconfigurable hydra-ulic system under consideration requires developing dynamic models for each of these two branches.

4. Results of Mathematical Modeling. The frequency characteristics were determined for two selected branches of the reconfigurable hydraulic system: from the starting tank to the gas generator $\frac{\delta \bar{p}_{\text{gg}}}{\delta \bar{p}_{\text{B}}}(j\,\omega)$ and from the fuel pump outlet to the gas generator $\frac{\delta \bar{p}_{\text{gg}}}{\delta \bar{p}_{\text{P}}}(j\,\omega)$. In evaluating these frequency characteristics, each check valve was assumed to be either fully open or fully closed. In the fully open state, the valve provides minimal hydraulic resistance; in the fully closed state, the hydraulic line containing the check valve is blocked. The boundary condition at the gas generator cross-section (impedance $\frac{\delta \bar{p}_{gg}}{\delta \bar{\dot{m}}_{fgg}}(j\omega)$) was specified using the equations for determining the gas-generator pressure under choked discharge of the combustion products.

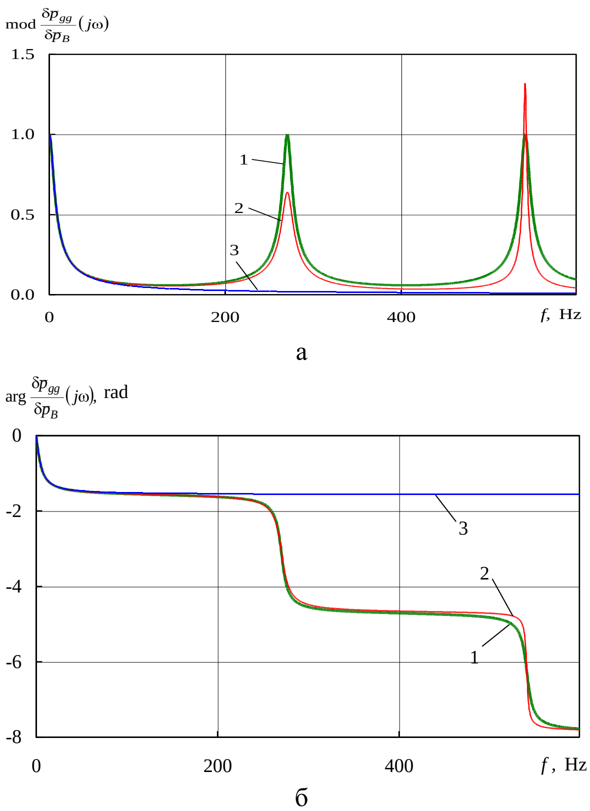
$$\frac{V_{gg}}{\gamma_{gg} R_{gg} T_{gg}} \frac{dp_{gg}}{dt} = \dot{m}_{ogg} + \dot{m}_{fgg} - \dot{m}_{sgg}, \quad \dot{m}_{sgg} = \frac{A_{gg} p_{gg}}{\sqrt{R_{gg} T_{gg}}} \sqrt{\gamma_{gg} \left(\frac{2}{\gamma_{gg}+1} \right)^{\frac{\gamma_{gg}+1}{\gamma_{gg}-1}}} . \quad (2)$$

Assuming $\dot{m}_{ogg} = const$ and $R_{gg} T_{gg} = const$, we obtain, where V_{gg} – is the volume of the gas generator combustion zone; R_{gg} – is the specific gas constant of the combustion products in the gas generator; T_{gg} – is the temperature of the

$$\left[\frac{V_{gg}}{\gamma_{gg} R_{gg} T_{gg}} j\omega + \frac{A_{gg}}{\sqrt{R_{gg} T_{gg}}} \sqrt{\gamma_{gg} \left(\frac{2}{\gamma_{gg}+1} \right)^{\frac{\gamma_{gg}+1}{\gamma_{gg}-1}}} \right] \delta p_{gg} = \delta \dot{m}_{fgg}$$

combustion products in the gas generator; γ_{gg} – is the specific heat ratio of the combustion products in the gas generator; A_{gg} – is the critical (throat) area of the gas generator.

The frequency dependences of the gain factors $\frac{\delta \bar{p}_{gg}}{\delta \bar{p}_B}(j\omega)$ and $\frac{\delta \bar{p}_{gg}}{\delta \bar{p}_P}(j\omega)$ for the distributed-parameter mo-dels are shown in Figs. 2 and 3. They were obtai-ned under boundary conditions corres-ponding to the nominal propellant mixture ratio in the gas generator. The frequency range over which gain factors $\frac{\delta \bar{p}_{gg}}{\delta \bar{p}_B}(j\omega)$ and $\frac{\delta \bar{p}_{gg}}{\delta \bar{p}_P}(j\omega)$ are presented was selected to include at least one resonant frequency.



1 – distributed-parameter model; 2 – lumped-parameter model; 3 – lumped-parameter model with $C_j = 0$

Fig. 3 – Magnitude (a) and phase (b) of the gain factor $\frac{\delta \bar{p}_{gg}}{\delta \bar{p}_B}(j\omega)$

For the feedline from the pump, the resonant frequency is 459 Hz, whereas for the feedline from the starting tank, two resonant frequencies fall within the same range – 272 Hz and 541 Hz. This range was selected with a margin, because the working frequency range typically considered in mathematical modeling of low-frequency dynamic processes in LPREs does not exceed 100 Hz [4]. It should be noted that the "gas generator-starting tank" line exhibits a larger number of resonant frequencies, which is attributed to its greater length.

When constructing the lumped-parameter dynamic model of the hydraulic system, the large length of the "gas generator–starting tank" line was taken into account. For this purpose, this line was divided into three sections and therefore includes two intermediate nodes (nodes p_1 and p_2 in Fig. 1). Another node is located at the branching point (p_3). Lumped compliances C_1, C_2 and C_3. are placed at these nodes. In the pipelines between the nodes, the liquid is treated as incompressible and its dynamics are described by the momentum equations. The hydraulic resistances introduced by the check valves are determined by their effective flow areas and depend primarily on the pressure drop between the valve inlet and outlet.

According to the proposed approach, the lumped-parameter dynamic model of the reconfigurable fuel hydraulic system can be written as follows:

$$J_{\text{fb}} \frac{d\dot{m}_{\text{fb}}}{dt} = p_{\text{B}} - p_1 - \xi_{\text{fb}} / \rho \cdot \dot{m}_{\text{fb}}^2, \tag{3}$$

$$C_1 \frac{dp_1}{dt} = \dot{m}_{\text{fb}} - \dot{m}_{\text{f12}}, \tag{4}$$

$$J_{\text{f12}} \frac{d\dot{m}_{\text{f12}}}{dt} = p_1 - p_2 - \xi_{\text{f12}} / \rho \cdot \dot{m}_{\text{f12}}^2, \tag{5}$$

$$C_2 \frac{dp_2}{dt} = \dot{m}_{\text{f12}} - \dot{m}_{\text{f23}}, \tag{6}$$

$$J_{\text{f23}} \frac{d\dot{m}_{\text{f23}}}{dt} = p_2 - p_3 - \left(\xi_{\text{f23}} + \xi_{\text{V1}}\right) / \rho \cdot \dot{m}_{\text{f23}}^2, \tag{7}$$

$$J_{\text{fp}} \frac{d\dot{m}_{\text{fp}}}{dt} = p_{\text{p}} - p_3 - \left(\xi_{\text{fp}} + \xi_{\text{V2}}\right) / \rho \cdot \dot{m}_{\text{fp}}^2, \tag{8}$$

$$C_3 \frac{dp_3}{dt} = \dot{m}_{\text{f23}} + \dot{m}_{\text{fp}} - \dot{m}_{\text{fgg}}, \tag{9}$$

$$J_{\text{fgg}} \frac{d\dot{m}_{\text{fgg}}}{dt} = p_3 - p_{\text{gg}} - \left(\xi_{\text{fgg}} + \xi_{\text{V3}}\right) / \rho \cdot \dot{m}_{\text{fgg}}^2, \tag{10}$$

where ρ – is the fuel density; ξ_{fb}, ξ_{f12}, ξ_{f23}, ξ_{fp}, ξ_{fgg} – are hydraulic resistance coefficients determined from hydraulic calculations; J_{fb}, J_{f12}, J_{f23}, J_{fp}, J_{fgg} – are inertial resistance coefficients obtained from the geometric characteristics of the pipelines; ξ_{V1}, ξ_{V2}, ξ_{V3} – are the hydraulic resistance coefficients of the check valves, determined by their effective flow areas A_{V1}, A_{V2}, A_{V3}:

$$\xi_{\text{V1}} = \frac{1}{2A_{\text{V1}}},\ \xi_{\text{V2}} = \frac{1}{2A_{\text{V2}}},\ \xi_{\text{V3}} = \frac{1}{2A_{\text{V3}}}. \tag{11}$$

Thus, the dynamic behavior of the analyzed lumped-parameter reconfigurable hydraulic system is described by a system consisting of the ordinary differential equations (3)–(10), the expressions for the hydraulic resistance coefficients of the check valves (11), and the gas-generator boundary conditions (2).

The values of the lumped compliances C_1, C_2 and C_3 are selected to achieve the required accuracy in matching the frequency characteristics of the hydraulic system obtained from the distributed-parameter and lumped-parameter models. Figures 2 and 3 show the gain factors of the lumped-parameter hydraulic system $\frac{\delta\bar{p}_{\text{gg}}}{\delta\bar{p}_{\text{B}}}(j\,\omega)$ and $\frac{\delta\bar{p}_{\text{gg}}}{\delta\bar{p}_{\text{P}}}(j\,\omega)$, obtained with the same compliance values C_1, C_2 and C_3, used for both gain factors. An analysis of Figs. 2 and 3 indicates that the gain factors predicted by the distributed-parameter and lumped-parameter models are in satisfactory agreement over the considered frequency range. In addition, it was found that the gain factors $\frac{\delta\bar{p}_{\text{gg}}}{\delta\bar{p}_{\text{B}}}(j\,\omega)$ and $\frac{\delta\bar{p}_{\text{gg}}}{\delta\bar{p}_{\text{P}}}(j\,\omega)$ obtained under different boundary conditions and different propellant mixture ratios in the gas generator, can also be described satisfactorily by the lumped-parameter system using the same compliance values C_1, C_2 and C_3. This makes it possible to use constant compliance values , and C_3 throughout the entire engine start-up process.

Introducing compliances C_1 and C_2 into the lumped-parameter model made it possible to reproduce two resonances in the frequency response $\frac{\delta\bar{p}_{\text{gg}}}{\delta\bar{p}_{\text{B}}}(j\,\omega)$. The introduced compliance values were set equal to the compliance at the pipeline branching node.

$$\tilde{C}_2 = \frac{C_2}{C_3} = 1.04, \qquad \tilde{C}_1 = \frac{C_1}{C_3} = 0.65.$$

Figures 2 and 3 also present the gain factors of the lumped-parameter hydraulic system $\frac{\delta\bar{p}_{\text{gg}}}{\delta\bar{p}_{\text{B}}}(j\,\omega)$ and $\frac{\delta\bar{p}_{\text{gg}}}{\delta\bar{p}_{\text{P}}}(j\,\omega)$ obtained with zero values of the lumped compliances C_1, and . These results indicate that, for the lumped-parameter model describes the dynamic processes satisfactorily within the frequency range up to 70 Hz. This model is simpler and can be used for mathematical modeling of low-frequency dynamic processes; however, in this case the system equations take a slightly different form. The absence of compliances implies equality of the flow rates at the node. Therefore, to determine the nodal pressure, instead of the continuity equation it is necessary to use a set of momentum equations in which the time derivatives of the flow rates are set to zero.

Conclusions.

1. A methodological approach to the mathematical modeling of dynamic processes in a branched reconfigurable hydraulic system has been developed. First, the frequency characteristics of several configurations of the reconfigurable hydraulic system occurring at different stages of its operation are determined using a distributed-parameter model. Next, a lumped-parameter mathematical model is formulated, incorporating the corresponding fluid momentum and continuity equations. The inertial resistance coefficients are determined from the geometric characteristics of the hydraulic lines, whereas the coefficients of hydraulic resistance

are obtained from engine balance calculations or from the corresponding pressure-drop relations for hydraulic losses. Lumped compliances are placed in the computational hydraulic scheme, typically at pipeline branching nodes or at locations associated with long pipeline sections. Their number and values are selected so that, for each configuration of the reconfigurable hydraulic system, the frequency characteristics of the distributed-parameter and lumped-parameter models match within a prescribed accuracy over a specified frequency range.

2. To demonstrate the proposed approach, a reconfigurable hydraulic system was considered, namely the fuel feed system of the gas generator of a test restartable LPRE employing a gas-generator afterburning (staged-combustion) cycle. Two configurations of the hydraulic system were identified: from the starting tank to the gas generator, and from the fuel pump outlet to the gas generator. For each configuration, the frequency characteristics were determined using distributed-parameter models. One resonant frequency of 459 Hz was identified in the pump line, whereas two resonant frequencies, 272 Hz and 541 Hz, were found in the longer line from the starting tank. A lumped-parameter model of the dynamic processes in the analyzed hydraulic system was then developed. It includes one node at the pipeline branching point and two nodes along the long pipeline between the starting tank and the gas generator. The values of the lumped compliances at the nodes were determined to achieve satisfactory agreement between the frequency characteristics of the distributed-parameter and lumped-parameter models. It was found that the lumped compliance values remain virtually unchanged under different system configurations, boundary conditions, and propellant mixture ratios in the gas generator.

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